SF2729 Groups and Rings
Problem set 7

due: Monday Jan 27 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Using the Sylow theorems, show that a group of order 132 must have a normal subgroup.

Problem 2. On the set $\mathcal{P}(X)$ of subsets of a set X, define an addition by

$$S + T = (S \cup T) - (S \cap T)$$

and a multiplication by

$$S \times T = S \cap T.$$

Show that with this structure, $\mathcal{P}(X)$ is a commutative ring with identity.

Problem 3. Let $R_1 = \mathbf{Z}[\sqrt{2}]$ and $R_2 = \mathbf{Z}[\sqrt{-2}]$. Compute the group of units $R_1^\times$, $R_2^\times$ of R_1 and R_2.