SF2729 Groups and Rings
Problem set 8

due: Monday Feb 3 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Denote by R the set of all polynomials in $f(x) \in \mathbf{Q}[x]$ with the property that $f(n) \in \mathbf{Z}$ for all $n \in \mathbf{Z}$. Show that R is a unital subring of $\mathbf{Q}[x]$ and give an example of a polynomial in R which is not in $\mathbf{Z}[x]$.

Problem 2. Given a homomorphism of commutative rings $\phi\colon R \to S$, show that

(1) if $I \lhd R$ is an ideal then $\phi(I) < S$ is an ideal in $\phi(R)$ but not necessarily in all of S. (Give a counterexample for the latter statement.)
(2) if $J \lhd S$ is an ideal then $\phi^{-1}(J) \lhd R$ is an ideal.

Problem 3. Let R be a finite commutative ring with unity. Show that every prime ideal in R is maximal.