SF2729 Groups and Rings
Problem set 9

due: Monday Feb 10 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Show that every ideal in the polynomial ring $\mathbf{Q}[x]$ is generated by a single element.

Problem 2. Let p be a prime number and $S = \{n \in \mathbf{N} \mid p \nmid n\} \subset \mathbf{Z}$. Show that S satisfies the conditions of Theorem 15 (p. 261). Denote the ring $S^{-1}\mathbf{Z}$ by $\mathbf{Z}_{(p)}$. Show that $\mathbf{Z}_{(p)}$ has exactly one nonzero prime ideal.

Problem 3. Using the Chinese Remainder Theorem, find all integer solutions to the system of congruences

$$x \equiv 3 \pmod 4, \quad x \equiv -2 \pmod{15}, \quad x \equiv 4 \pmod 7.$$