SF1611 Introductory course in mathematics I. 1.5 cr
Sample exam. Duration: 60 minutes. No aids allowed

The problems are worth 1 credit each and you are only required to provide answers, not complete derivations. In order to pass, you must get at least 5 credits.

Name:..Pers.no....................................Program.............

Result:

1	2	3	4	5	6	7	8	Σ	**Grade**

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1. Write in words how the following statement is pronounced.

$$\forall\delta > 0\ \exists\varepsilon > 0,\ |x| < \varepsilon\ \Rightarrow\ |\sin x| < \delta$$

Answer:

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2. Use mathematical symbols to define the set of all real numbers whose distance from 7 on the number line is strictly greater than 2.
Answer:

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3. The third-degree polynomial $x^3 - 4x^2 - 7x + 10$ has a zero at $x = 1$. Find the remaining zeros.
Answer:

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4. Find all positive solutions to the equation $x - 2 = \sqrt{x}$.
Answer:

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5. Find an integer n such that $|\frac{n}{5} - e| < \pi - 3$.
Answer:

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6. Simplify $e^{2\ln 2}$ as much as possible.
   Answer:

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7. Find all real solutions to the equation $\cos 2x = 1/2$.
   Answer:

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8. The Fibonacci number sequence $f_0, f_1, f_2, \ldots$ begins with 0 and 1 and after that every entry is the sum of the two previous ones:

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots$$

Fill in the gap in the following proof that $f_n < 2^n$ for any natural number n.

Induction over n. For $n = 0$ and $n = 1$ the statement is true since $f_0 = 0 < 1 = 2^0$ and $f_1 = 1 < 2 = 2^1$. Under the assumption that the statement holds for $n - 1$ and $n - 2$ we want to show that it holds also for n.

By definition $f_n = f_{n-1} + f_{n-2}$, and by the induction assumption we have

Thus

$$f_n = f_{n-1} + f_{n-2} < 2^{n-1} + 2^{n-2} < 2^{n-1} + 2^{n-1} = 2^n.$$