KTH Teknikvetenskap

SF2729 Groups and Rings
Exam
Monday, March 16, 2015

Time: 08:00-13:00
Allowed aids: none
Examiner: Wojciech Chachólski

Present your solutions to the problems in a way such that arguments and calculations are easy to follow. Provide detailed arguments to your answers. An answer without explanation will be given no points.

The final exam consists of six problems, each of which can give up to 6 points, for a sum of 36 points.

The final score is the better of the final exam score; and the weighted average of the final exam score (75%) and the homework score (25%). The minimum scores required for each grade are given by the following table:

Grade	A	B	C	D	E	Fx
Credit	30	27	24	21	18	16

A score of 16 or 17 is a failing grade with the possibility to improve to an E grade by additional work.

Problem 1. (6 points).

Let $\sigma = (147)(2356) \in S_7$, the symmetric group on the letters $\{1, \dots, 7\}$.

(a) What is the order of σ?

(b) What is the centralizer $C_{S_7}(\sigma)$?

(c) How many elements of the same order as σ are there in S_7?

Problem 2. (6 points).

Let n be a positive natural number. What is the number of solutions of the equation $x^2 - x = 0$ in $\mathbb{Z}/n\mathbb{Z}$? You might first try to solve the case where n is prime power and then use the Chinese Remainder Theorem.

Problem 3. (6 points).

Let $f\colon G \to H$ be a group homomorphism with kernel K and image I. Show:

(a) For every subgroup $N \leq G$, $f^{-1}(f(N)) = KN = \{kn \mid k \in K, n \in N\}$.

(b) For every subgroup $L \leq H$, $f(f^{-1}(L)) = I \cap L$.

Problem 4. (6 points).

Let R be a PID and $S \subset R$ be a multiplicative subset. Show that $R[S^{-1}]$ is also a PID.

Problem 5. (6 points).

Let G be a group of order $637 = 7^2 \cdot 13$. Show that G is abelian.

Problem 6. (6 points).

Let R be the ring $\mathbb{Z}[\sqrt{-2}]$. Recall that R is a Euclidean domain with Euclidean multiplicative norm $N(a + b\sqrt{-2}) = a^2 + 2b^2$.

(a) Prove that $1 + 2\sqrt{-2}$ is a reducible element of R.

(b) Determine a greatest common divisor in R of $2 + \sqrt{-2}$ and $4 + \sqrt{-2}$.

(c) Prove that $R/(3 + \sqrt{-2})$ is a finite field with 11 elements.