Problem73# 2.5 Solution

Case $n = 3$:

Assume $x_j > 0$.

$$\frac{x_1}{x_2+x_3} + \frac{x_2}{x_3+x_1} + \frac{x_3}{x_1+x_2} \geq \frac{3}{2}$$

$$\Leftrightarrow \; \frac{2x_1}{x_2+x_3} + \frac{2x_2}{x_3+x_1} + \frac{2x_3}{x_1+x_2} - 3 \geq 0$$

$$\Leftrightarrow \; \frac{2x_1 - x_2 - x_3}{x_2+x_3} + \frac{2x_2 - x_3 - x_1}{x_3+x_1} + \frac{2x_3 - x_1 - x_2}{x_1+x_2} \geq 0$$

$$\Leftrightarrow \; \frac{x_1 - x_2 + x_1 - x_3}{x_2+x_3} + \frac{x_2 - x_3 + x_2 - x_1}{x_3+x_1} + \frac{x_3 - x_1 + x_3 - x_2}{x_1+x_2} \geq 0$$

$$\Leftrightarrow \; (x_3+x_1)(x_1^2 - x_2^2) + (x_1+x_2)(x_1^2 - x_3^2) + (x_1+x_2)(x_2^2 - x_3^2) + \\ (x_2+x_3)(x_2^2 - x_1^2) + (x_2+x_3)(x_3^2 - x_1^2) + (x_3+x_2)(x_3^2 - x_1^2) \geq 0$$

$$\Leftrightarrow \; (x_1 - x_2)(x_1^2 - x_2^2) + (x_1 - x_3)(x_1^2 - x_3^2) + (x_2 - x_3)(x_2^2 - x_3^2) \geq 0$$

$$\Leftrightarrow \; (x_1 - x_2)^2(x_1 + x_2) + (x_1 - x_3)^2(x_1 + x_3) + (x_2 - x_3)^2(x_2 + x_3) \geq 0$$

which is obvious.
Equality occurs only if $x_1 = x_2 = x_3$.

BE/GJ