**Problem73# 2.7**

To prove:

(*) $x^\lambda(x-y)(x-z) + y^\lambda(y-z)(y-x) + z^\lambda(z-x)(z-y) \geq 0.$
$(x, y, z > 0,\ \lambda$ real) .

Note first that <u>any</u> permutation of $x, y, z$ leaves the left hand side function invariant. So we can assume wlog that $x \geq y \geq z (> 0)$.
Then rewrite the LHS (now with nonnegative factors) as:
$x^\lambda(x-y)(x-z) - y^\lambda(y-z)(x-y) + z^\lambda(x-z)(y-z) = t_1 - t_2 + t_3.$
(all $t_j \geq 0$).
Assume $\lambda \geq 0$ and compare factors of $t_1$ and $t_2$ : $x^\lambda \geq y^\lambda$ and $(x-z) \geq (y-z)$.
Hence $t_1 \geq t_2$ and (*) holds.
Then assume $\lambda < 0$.
Compare factors of $t_3$ and $t_2$ : $z^\lambda \geq y^\lambda$ and $(x-z) \geq (x-y)$.
Hence $t_3 \geq t_2$ and (*) holds also for negative $\lambda$. QED.

The proof shows also that equality occurs only if $x = y = z$.
(E.g in case $\lambda \geq 0$, $t_1 = t_2$ only if $x = y$ and $t_3 = 0$ only if, in addition, $z = x = y$. )