KTH Matematik

5B1456, , Matrix Algebra
Assignment 3
due Sep. 27, 2004

(1) Let V be a finite dimensional vector space over a field K. Choose a basis β of V and define the map:

$$Tr : Hom_K(V,V) \to K, \;\; Tr(T) = Trace([T]_\beta)$$

where $[T]_\beta \in M_n(K)$ is the matrix associated to T with respect to the basis β.

(a) Show that $Tr(AB) = Tr(BA)$.

(b) Prove that the definition of Tr is independent on the choice of the basis and that Tr is a K-linear transformation. Is it a ring-homomorphism?

(2) Let L be a field and $K \hookrightarrow L$ be a subfield such that L is a finite dimensional vector space over K. For every $e \in L$ denote by the same symbol the multiplication map

$$e : L \to L, \quad e(x) = ex.$$

It is a K-linear transformation.

(a) Show that $B : L \times L \to K$, defined as $B(e_1, e_2) = Tr(e_1 e_2)$ is a bilinear form.

(b) Write the form explicitly for $\mathbb{R} \hookrightarrow \mathbb{C}$.

(c) Prove that if char(K)=0 then B is non-degenerate.