Introduction to Game Theory
Problem Set #2

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1. Consider the two-player game

	L	R
T	$2,7$	$0,0$
B	$2,0$	$3,1$

(a) Find the set of (pure and mixed) Nash equilibria.

(b) Which of the Nash equilibria, if any, are weakly undominated?

(c) Which of the Nash equilibria, if any, are perfect, proper, essential?

2. Consider the two-player game

	L	C	R
T	$5,5$	$3,0$	$0,2$
M	$5,1$	$2,1$	$1,0$
B	$0,0$	$2,5$	$4,2$

(a) Find its pure-strategy Nash equilibria.

(b) For each player, find the set of pure strategies that are *not* iteratively strictly dominated (by any pure or mixed strategy).

(c) Find all pure-strategy perfect equilibria.

3. Consider the two-player game

	L	C	R
T	$7,7$	$1,7$	$1,0$
M	$7,1$	$5,5$	$0,2$
B	$0,1$	$2,0$	$6,0$

(a) Find its set of pure-strategy Nash equilibria.

(b) Find its set of pure-strategy perfect equilibria.

(c) Find the set of iteratively strictly dominated pure strategies for each player.

(d) Delete all iteratively strictly dominated pure strategies from the game, and do tasks (a)-(b) for the so reduced game.

4. Consider two firms in price competition in a market for a homogeneous good. The firms simultaneously choose their prices, p_1 and p_2, from a given set $P \subset \mathbb{R}_+$. All customers buy from the firm with the lowest price. If both firms set the same price, then each firm receives half of the market demand at that price. Both firms produce at a constant unit cost, c_1 and c_2, respectively, and have no fixed costs. Let market demand be $D(p) \equiv a - p$ for some $a > 0$, and assume that $0 \leq c_1 \leq c_2 < a$.

   (a) For each firm, find its monopoly price; the price it would have set, had it been alone in the market.

   (b) For $c_1 = c_2$ and $P = [0, a]$: Define the associated normal-form game (strategy sets and payoff functions). Are the payoff functions continuous? Do best replies always exist? Does there exist Nash equilibria in pure strategies? Does there exist any pure-strategy Nash equilibrium in undominated strategies?

   (c) For $c_1 < c_2 < a$ and $P = [0, a]$: The same questions as in (b).

   (d) Do (b) and (c) for $P = \{0, 1, 2, ..., a\}$ when $0 < c_1 \leq c_2 < a$ are integers.

   (e) In the same setting as (d): Find the sets of perfect equilibria, proper equilibria and essential equilibria. Find all strategically stable sets.

5. Consider two firms in a market for a homogeneous good. They simultaneously choose their output quantities, $q_1 \in [0, a]$ and $q_1 \in [0, a]$, and thereafter the market clears, resulting in the market price $p = a - q_1 - q_2$. Each firm produces under constant marginal cost, $c_i \in \{c^L, c^H\}$, where $0 \leq c^L < c^H < a$, and thus the profit to each firm is $\pi = (a - q_1 - q_2 - c_i)\, q_i$. Each firm only knows its own marginal cost, and strives to maximize its expected profit, conditional upon its own type. Model this strategic interaction as a game between "nature" (player 0), firm 1 (player 1) and firm 2 (player 2), where nature first selects a pair of marginal costs, (c_1, c_2), according to a probability distribution $\mu = (\mu_{LL}, \mu_{LH}, \mu_{HL}, \mu_{HH})$ for the four possible cost-combinations, then each firm learns its own cost (only), and thereafter both firms simultaneously choose their output levels. Both firms know the probability distribution μ.

   (a) Draw an extensive-form representation for this interaction.

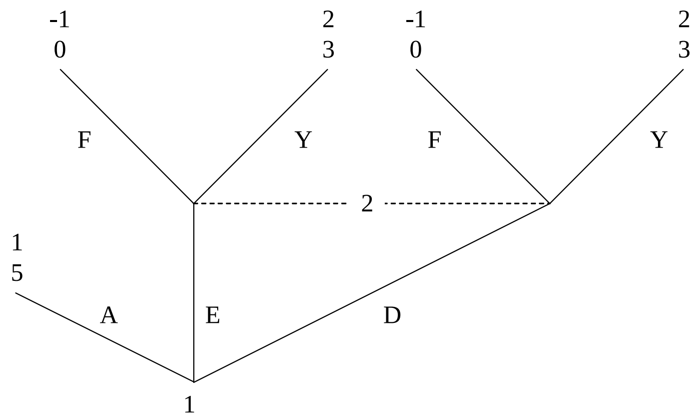
   (b) Define the associated normal form game (pure-strategy sets, payoff functions).

   (c) Suppose that the marginal cost of firm 1 is c^L for sure: $\mu_{HL} = \mu_{HH} = 0$. Find all Nash equilibria in pure strategies.

   (d) Suppose that the marginal costs are statistically independent: $\mu_{LL} = \lambda^2, \mu_{LH} = \mu_{HL} = (1 - \lambda)\lambda, \mu_{HH} = (1 - \lambda)^2$ for some $\lambda \in (0, 1)$. Find all Nash equilibria in pure strategies.

   (e) Suppose that the marginal costs are perfectly positively correlated: $\mu_{LH} = \mu_{HL} = 0$. Find all Nash equilibria in pure strategies.

6. Consider the entry deterrence game Γ given below, in which player 1 is a potential entrant, who can enter in two payoff-equivalent ways, E or D. Player 2 is a monopolist, who can "fight" or "yield" upon 1's entry.



(a) Write up the (pure-strategy) normal form G of Γ. Find all Nash equilibria of the mixed-strategy extension, $\tilde{G}$.

(b) In $\tilde{G}$: Find all perfect, proper and essential equilibria of $\tilde{G}$.

(c) In $\tilde{G}$: Find all essential Nash-equilibrium components

(d) In Γ: Find all subgame perfect equilibria and all sequential equilibria.