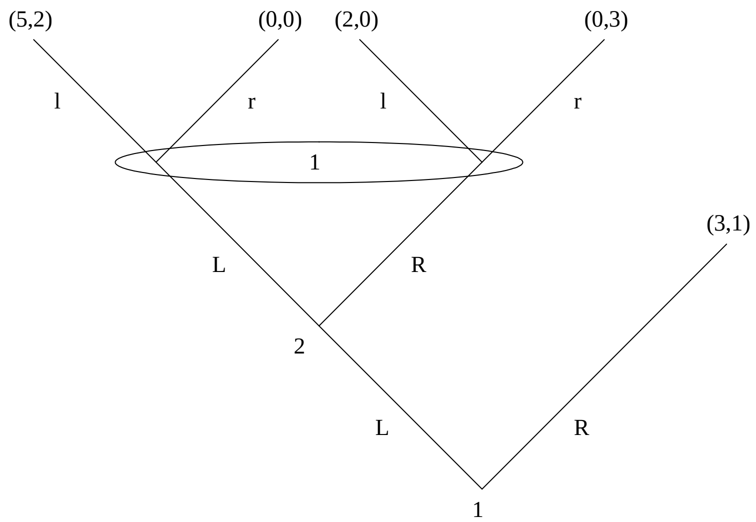
Introduction to Game Theory
Problem Set #3

Jörgen Weibull

1. Consider the extensive-form game



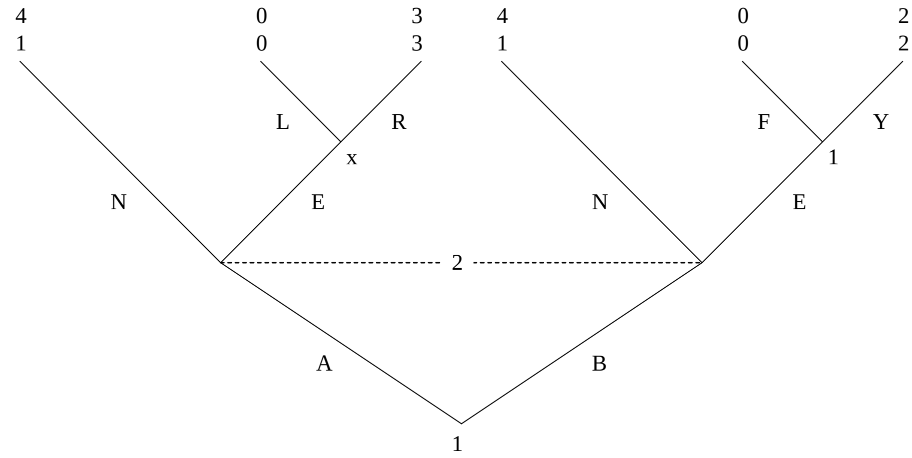
(a) Find the set of pure-strategy Nash equilibria

(b) Find the set of subgame perfect equilibria

(c) Find the set of sequential equilibria

(d) Find the set of NF and EF perfect equilibria, respectively.

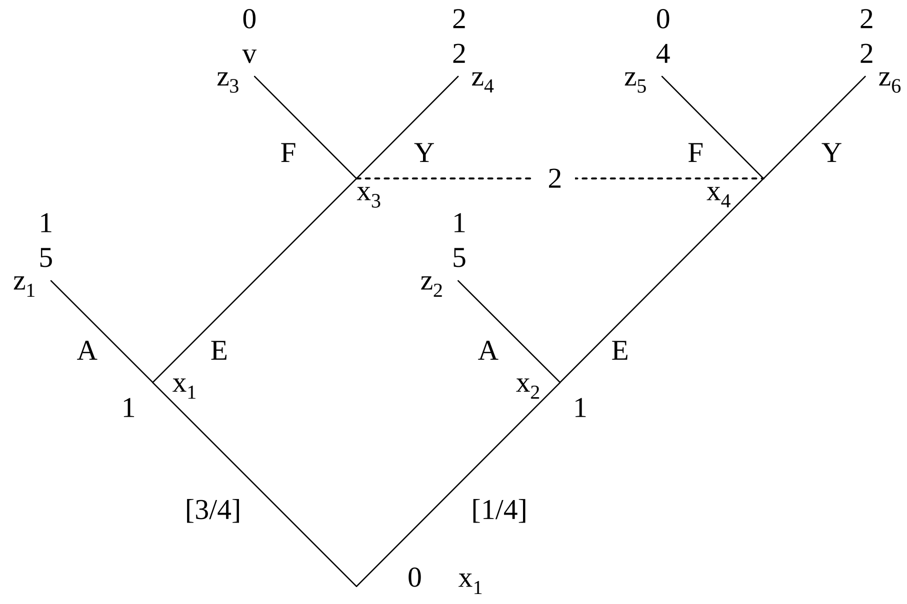
2. Consider the extensive-form game



(a) Suppose node x is "owned" by player 1. Is the pure strategy profile where 1 plays BLY and 2 plays E a Nash equilibrium, a sequential equilibrium?

(b*)* Suppose node x is instead "owned" by player 2. Is the pure strategy profile where 1 plays BY and 2 plays EL a Nash equilibrium, a sequential equilibrium?

3. Consider the game



For each payoff value $v > 0$:

(a) Find its set of Nash equilibria.

(b) Find its set of subgame perfect equilibria.

(c) Find its set of sequential equilibria.

4. Find all evolutionarily stable strategies in the following two games

$$G_1 = \begin{array}{ccc} & L & R \\ L & 1,1 & 0,0 \\ R & 0,0 & 3,3 \end{array} \qquad G_2 = \begin{array}{ccc} & L & R \\ L & 0,0 & 3,1 \\ R & 1,3 & 0,0 \end{array}$$

5. Consider the games

$$G_3 = \begin{array}{ccc} & L & R \\ T & 5,5 & 0,1 \\ B & 5,2 & 2,2 \end{array} \qquad \text{and} \qquad G_4 = \begin{array}{ccc} & L & R \\ T & 0,0 & 2,2 \\ B & 2,2 & 0,0 \end{array}$$

(a) Find all (pure and mixed) perfect equilibria in each game

(b) Does the concept of evolutionary stability apply to the two games? Find all evolutionarily stable strategies in the game(s) where the concept applies.

(c) For each of the two games, write up a meta-game in which nature first chooses the player roles, such that evolutionary stability applies to the meta-game, and find all evolutionarily stable strategies.

6. Consider the coordination game

$$\begin{array}{ccc} & a & b \\ a & 2,2 & 0,0 \\ b & 0,0 & 1,1 \end{array}$$

Suppose that the two players can costlessly exchange messages before playing this game. Let the set of messages, M, be finite, and assume that first both players simultaneously select a message to send, then they both observe the sent messages, and thereafter they simultaneously select an action, a or b, in the game above.

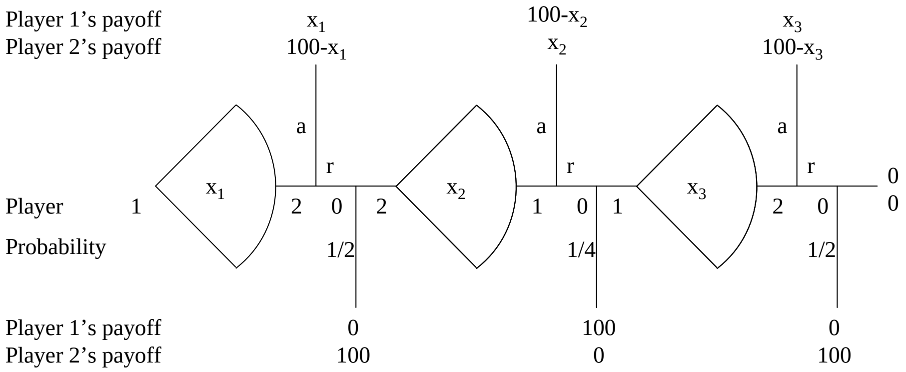
(a) Write up the (pure-strategy) normal-form of this cheap-talk game.

(b) Find a pure strategy in the cheap-talk game that is evolutionarily stable (in the cheap-talk game) and that results in the play of (a, a).

(c) Show that there exists no pure strategy in the cheap-talk game that is evolutionarily stable (in the cheap-talk game) and that results in the play of (b, b).

(d) Find a mixed strategy in the cheap-talk game that is evolutionarily stable (in the cheap-talk game) and that results in the play of (b, b).

7. The diagram below shows the (infinite-action) extensive form of a bargaining game.



There is a total payoff of 100 to be divided between players 1 and 2. But also "nature" (player 0) has three moves in the game. Player 1 begins the game by demanding some payoff $x_1 \in [0, 100]$ for herself. If player 2 accepts this demand ("a"), then the game ends, and the two players receive x_1 and $100 - x_1$ respectively. If player 2 instead rejects the demand ("r"), then Nature makes her first move. With probability 1/2 this ends the game, with probability 1/2 player 2 is given the opportunity to make a demand $x_2 \in [0, 100]$, etc. Note that if player 2 rejects 1's final demand, x_3, then Nature may end the game by giving payoff zero to both players.

(a) Find a subgame perfect equilibrium of this game. Is it unique?

(b) Find a Nash equilibrium with an outcome that is incompatible with subgame perfection.