# Introductory Game Theory

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# Power Allocation Based on Power Efficiency in Uplink OFDMA Systems: A Game Theoretic Approach

### *D. Yu, D. Wu, Y. Cai and W. Zhong*

Here a telecommunication system consisting of base stations and mobile users is considered in uplink, e.g., it is only considered that the mobile users transmit information to the base stations. In general the transmission takes place over a range of frequencies, known as the frequency band. In the considered OFDMA setting, the transmission takes place over a set of subcarriers, where each subcarrier is a part of the frequency band. The idea behind this approach is to limit the interference from close user, by connecting them to the same cell. The only interference with this setting is between users, connected to different base stations, using the same channel. Within this paper by Yu, et al., one single cell is consider, i.e., only one base station. Hence, there is no intra-cell interference, and hence no interference at all. The aspect of interest is to use the powers efficiently, when performing the channel assignment and power allocation. So, there is still a competition about the available resources, (i.e., which channels to use and how much power to use on those channels), though no interference is involved. Within the paper they propose two different games, where the second game is a modification of the first game where an additional pricing factor has been introduced (which makes the users cooperate). The objective of the game is to maximize the power efficiency, and they prove that their proposed game has at least one NE point. Based on this game theory knowledge they propose an algorithm to solve the subcarrier and power allocation problem.

# Subcarrier and Power Allocation in Uplink OFDMA Systems Based on Game Theory

### *D. Wu, D. Yu and Y. Cai*

Also here a subcarrier and power allocation system in uplink OFDMA is considered. The aim is to maximize the rate-sum capacity, i.e. the throughput of the system. The approach here is to allocate each subcarrier to the user which ensures that the normalized channel gain reaches its maximum. Given that all subcarriers have been allocated, the remaining goal is then to maximize the rate of each user by power allocation. Each mobile user is given a pricing factor, which is multiplied with the total power used by the mobile. The overall utility is this pricing term added with a quadratic term, which is the difference between the given maximum throughput of the user and the achieved throughput of the user. Hence, the system aims to reach the maximum throughput of each user, while using as little power as possible. This objective, together with power and rate constraints defines the non-cooperative game of this paper, which is proved to have a unique NE which is globally optimal. An allocation algorithm, which can be used to solve the defined non-cooperative game, is also given.

# Game Theory, Theorem
## (used in the two previous papers)

A Nash equilibrium exists in a game G, with L subcarriers, where G = [L,{P_n},{u_n(.)}] if {P_n} is a nonempty, convex and compact subset of a finite space and the utility functions u_n(.) are quasi-concave (quasi-convex) continuous functions in P.

# Fairness vs. Efficiency: Comparison of Game Theoretic Criteria for OFDMA Scheduling
## *A. Ibing and H. Boche*

A brief idea of telecommunication network scheduling is to share limited resources among users, to benefit in terms of the considered objective. One aspect would be to go for an extremely fair system where one aims to maximize the user worst off. Though the resources are evenly distributed, the downside of this approach is that the user with worst channel quality limits the entire system performance. Another extreme is to maximize the overall system efficiency, by instead maximize the total throughput, i.e., totally ignore the users worst off and only focus on the best users. In the paper "Fairness vs. Efficiency: Comparison of Game Theoretic Criteria for OFDMA Scheduling", by A. Ibing and H. Boche a smaller study in this direction is performed with a Game Theoretic approach. Part from the two extreme cases, a proportional fairness and a Kalai-Smorodinsky fair scheduling is performed. These two approaches ensure that each user is guaranteed some share of the resources, while better users are given a larger share. A smaller study is made on these four approaches, where proportional fairness and Kalai-Smorodinsky fairness perform most similar, while the two extremes maximize the total throughput and to maximize the minimum, end up on each side of the other two. Each of these four approaches are Pareto efficient, which is known from the book "Handbook of Game Theory, vol. 2, chapter 35: Cooperative Models of Bargaining" by W. Thomson. So in the end, it is up to the operator to choose which trade off between fairness and capacity that is suitable for the considered system.

## *References to the summarized papers above*

Power Allocation Based on Power Efficiency in Uplink OFDMA Systems: A Game Theoretic Approach, D. Yu, D. Wu, Y. Cai and W. Zhong, ICCS 2008.

Subcarrier and Power Allocation in Uplink OFDMA Systems Based on Game Theory, D. Yu, D. Wu and Y. Cai, IEEE Int. Conference Neural Networks & Signal Processing 2008.

Fairness vs. efficiency: Comparison of game theoretic criteria for OFDMA scheduling, A. Ibing and H. Boche, IEEE Xplore 2007.

# Reflections, ideas and criticism

It has been interesting to see how game theory can be used and applied in a telecommunication network OFDMA setting, which is the setup I consider in my research. I must admit that I hoped for “more advanced” game theory models, like for example a potential game setting, and not “only proving” existence of a NE and an algorithm that converges to a NE. However, I still found the papers interesting to read, and they had some nice ideas which are good to know. For example the paper “Subcarrier and power allocation in uplink OFDMA systems based on game theory” nicely introduced the normalized channel gain, and I also found the considered game, in the same paper, fairly advanced to have a unique global optimum - which might be a useful feature. In fact, that the more advanced game theory models and concepts seem to be unused in this type of OFDMA settings perhaps increase the possibility of producing a contribution of my own.

Mikael Fallgren, 2010-05-18.