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# Projects in Financial derivatives SF2975, 2016

## General instructions

### Introduction

Each project is done in a group of 2-4 students with identical groups for both projects. You create your own group, and the name of the group members should be emailed to armerin@math.kth.se no later than **18 September 2016**. Your mail should follow the format:

```
Subject: SF2975-Project
Body:
Firstname Lastname
Firstname Lastname
Firstname Lastname
```

### Grading

On each report you will be given a score in the range 0-25. A score of at least **20 points is needed to pass** the assignment. In order to get the grading pass on the project part of the course you need to pass both projects. A report that is not graded pass initially has to be completed until it is graded pass.

#### Submission

The projects should be submitted no later than **16 October 2016**. The latest date for handing in a completion is **13 November 2016**. No new reports are accepted after these dates. It is only possible to complete a previously submitted report. Hand in your reports at the student expedition at the Math department or in the opposite black mail box. Ensure that your name, the course number, and the instructors name are on the front page. The Swedish post office can be relied upon for those unable to attend in person.

### The reports

You are encouraged to find information in books, scientific papers and on the internet in order to solve the assignments of the projects. When you do so, always give a clear reference to your source. We recommend the LaTeX document preparation system for typesetting your reports. LyX is a good alternative to Word.

**Report submission guidelines**

1. Submit one typed solution per group in paper form. Do not submit electronic or handwritten solutions. The pages must be stapled together. Use only one side of each page.
2. The front page must contain:
   (a) Names and e-mail addresses (in the header or footer).
   (b) A list of short comments to *all* the assignments.
3. Deeper reasoning and calculations should be well organized and presented in the following pages. Code and plots is placed last in the document.

**Submissions failing to comply will be ignored.**

# Project I

## Introduction

The goal with this project is to estimate the local volatility surface. First of all read the text 'The local volatility surface', which can be found on the homepage of the course.

## Assignments

(a) Describe the stochastic model for the stock price in a local volatility model and then explicitly derive the Dupire formula by going through the steps I-III as outlined in the text 'The local volatility surface'.

(b) Use call option prices from the two csv-files found at the homepage of the course to estimate two local volatility surfaces using Dupire's formula. Each row in the csv-files consists of data on the form

```
Time to maturity, Call option price, Strike price.
```

Set $r = 0$ and approximate the derivatives in the Dupire formula using finite differences.

(c) The method used in (b) can be numerically unstable. Describe any problems you might have had. Check the robustness of the approach by changing the prices and study how the local volatility surface changes.

(d) Discuss alternative approaches to finding the local volatility surface from Dupire's formula when there is only a finite set of call options prices. Implement one of your alternatives, still with $r = 0$, and compare the surfaces you get with the one you got from using finite differences.

# Project II

## Introduction

In this project you will use a procedure described by the Swedish financial supervisory authority Finansinspektionen to estimate a yield curve. Start by reading the document FFFS 2013:23. It is available on the homepage of the course and exists in both Swedish and in an English translation.

*Note:* There is a supplement correcting an error in a formula in the Swedish legal text (the formula in the English translation is correct).

## Assignments

(a) Describe clearly the process of getting from market data represented by the swap rates $par(t)$ to the discount rates $z(t)$ that is given in FFFS 2013:23 Chapter 2. See also Appendix 1 and 2. You should not just cite the formulas in the legal text, instead you should give a presentation of the material.

(b) Implement the procedure which you have described in (a) numerically for the currency SEK. Use the NASDAQ OMX Swap Fixing with maturity from 1 to 10 years as the market data and calculate $z(t)$ for $t = 1, 2, \ldots, 100$. Use linear extrapolation to estimate the rates $\widetilde{f}(t-1, t)$ between $T1$ and $T2$ years. You should present your output both in a table and in a figure.

(c) Now use the calculated discount factors from (b) to estimate an observed yield curve $y^{\star}(t)$ for every $t \in [0, 100]$. Do this by fitting the model

$$y(t) = \beta_0 + \beta_1 e^{-t/\tau} + \beta_2 t e^{-t/\tau}$$

to your data points from (b), i.e. find values of the parameters $\beta_0$, $\beta_1$, $\beta_2$ and $\tau$ that best fits your data. The relation between $z(t)$ and $y(t)$ is given by

$$y(t) = \ln(1 + z(t)),\ t = 1, 2, ..., 100.$$

(d) Finally use the estimated yield curve from (c) to calibrate a Hull-White model. The input in the calibration equation is the observed instantaneous forward rate curve $f^{\star}$ and the model parameters $a$ and $\sigma$. Calculate the observed forward rate curve from the observed yield curve by using the relation

$$f^{\star}(t) = y^{\star}(t) + t\frac{\partial y^{\star}(t)}{\partial t}.$$

Choose some suitable values on $a$ and $\sigma$ and use these values together with $f^{\star}$ to find the curve $\Theta$ given in the Hull-White model. How sensitive to the choice of $a$ and $\sigma$ is the function $\Theta$?

**Remarks**

- In the book an observed instantaneous forward curve, yield curve and an observed yield curve should have been denoted $f^\star(0,t)$, $y(0,t)$ and $y^\star(0,t)$ respectively, but in this project we always are at time 0, so we suppress the first argument.
- The forward rates $f(t-1,t)$ in thel text are *not* equal to the instantaneous forward rates in the book.