Avd. Matematisk statistik

KTH Matematik

TENTAMEN I 5B1508 MATEMATISK STATISTIK FÖR S
TISDAGEN DEN 20 DECEMBER 2005 KL 08.00–13.00.

Examinator: Gunnar Englund, tel. 790 7416.

Tillåtna hjälpmedel: Formel- och tabellsamling i Matematisk statistik. Räknare.

Införda beteckningar skall förklaras och definieras. Resonemang och uträkningar skall vara så utförliga och väl motiverade att de är lätta att följa. Numeriska svar skall anges med minst två siffrors noggrannhet. Varje korrekt lösning ger 10 poäng. Gränsen för godkänt är preliminärt 20 poäng.

Den som fått 18 eller 19 poäng på tentamen har möjlighet till komplettering. Kompletteringen skall göras inom tre veckor efter det att resultatet av denna tentamen publicerats. Den som är aktuell för komplettering skall till examinator anmäla önskan att få en sådan inom två veckor från publicering av tentamensresultatet.
Den som fått godkänt på lappskrivning nummer 1 från den 16 november 2005, får uppgift 1 a) tillgodoräknad. Den som fått godkänt på lappskrivning nummer 2 från den 30 november 2005, får uppgift 2 a) tillgodoräknad. Tillgodoräknade uppgifter skall inte lösas.

Resultatet anslås senast fredagen den 13 januari 2005 på Matematisk statistiks anslagstavla i entréplanet, Lindstedtsvägen 25, rakt fram innanför porten.

Tentamen kommer att finnas tillgänglig på elevexpeditionen sju veckor efter skrivningstillfället.

Uppgift 1

a) The independent events A, B and C satisfy $P(A) = 0.3$, $P(B) = 0.4$ and $P(C) = 0.1$. Calculate the probability that at least one of the events A, B and C occur. (6 p)

b) An elevator in KTH is marked "Elevator for at most 400 kg or at most 5 persons". A randomly selected male student's weight is a $N(75, 10)$-distributed random variable, an a randomly selected female student's weight is a $N(65, 5)$-distributed random variable. Assume that 3 male and 2 female students plan to use the elevator. All the weights of the students are assumed to be independent. Calculate the probability that their combined weight exceeds 400 kg. (4 p)

Uppgift 2

A certain type of microcircuits are delievered in batches of 1000 units. The following testing scheme is used: From each batch a random sample of 80 units are selected without replacement and tested. If at most three units are defective the batch is accepted – if not it is not accepted. Calculate

a) the probability that a batch is not accepted when the proportion defective units is 0.005. (6 p)

b) the probability that a batch is accepted when the proportion defective units is 0.15. (4 p)

Appropriate and well motivated approximations can be used in both the a- and b-parts.

Uppgift 3

In order to predict the sheer strength of steel girders two different methods are used, here called method 1 and method 2. In a comparative study the quotient between the predicted value and the real value was calculated for 5 different girders with the two methods. The values obtained were as follows:

Girder nr	1	2	3	4	5
Method 1	1.19	1.15	1.32	1.34	1.20
Method 2	1.06	0.99	1.06	1.06	1.07

Result k, $k = 1, \ldots, n$, for method 1 is seen as an observation from $\mathrm{N}(\mu_k + \Delta, \sigma_1)$ and result k from method 2 as an observation from $\mathrm{N}(\mu_k, \sigma_2)$. All random variables are assumed to be independent.

a) Calculate a 95% confidence intervall for the systematic difference Δ. (7 p)

b) Test the hypothesis $H_0 : \Delta = 0.25$ against the hypotheses $H_1 : \Delta \neq 0.25$ on the level 5%. It should be clearly stated if H_0 is rejected or not. (3 p)

Uppgift 4

In a paper in *Concrete Research*, Vol. 41, 1989, data are presented for the connection between pressure sustainability x och permeability y for different mixtures of concrete.
The data for $n = 14$ pairs (x_i, y_i) of observations is

$$\textstyle\sum_{i=1}^{14} y_i = 572 \qquad \sum_{i=1}^{14} y_i^2 = 23530$$

$$\textstyle\sum_{i=1}^{14} x_i = 43 \qquad \sum_{i=1}^{14} x_i^2 = 157.42$$

$$\textstyle\sum_{i=1}^{14} x_i y_i = 1697.80.$$

We assume that (x_1, y_1),...,(x_{14}, y_{14}) are related to each other according to a simple regression model, i.e. as observations from (x_1, Y_1),...,(x_{14}, Y_{14}), where each Y_i is normally distributed $N(\alpha + \beta x_i, \sigma)$

a) Calculate the estimates α^*_{obs}, β^*_{obs} of the coefficients in the regression line. (3 p)

b) Use the estimated regression line to estimate the expected value of the permeability when pressure sustainability x is 4.3. (2 p)

d) Calculate confidence intervals with confidence level 95% for α and β respectively. (5 p)

Uppgift 5

For a bit-stream over a noisy channel can bits (1:s and 0:s) be incorrectly transmitted. Assume that a bit (1 or 0) is incorrectly transmitted with probability p independently of other bits. Over channel A the error probability for a bit is 0.005 and over channel B the error probability is 0.015.
When a bit-stream of $n = 1000$ bits were transmitted, $x = 10$ bits were incorrectly transmitted.

a) Use the Maximum Likelihood-method to decide which channel (A or B) was used to transmit the bit-stream. (5 p)

b) It is known that 75% of all bit-streams use channel A, the rest use channel B. With

$$A = \{\text{Channel A is used}\} \quad \text{and} \quad B = \{\text{Channel B is used}\} = A^*$$

is $P(A) = 0.75$ and $P(B) = 0.25$.

Calculate $P(A|10 \text{ errors in } 1000 \text{ bits})$ and $P(B|10 \text{ errors in } 1000 \text{ bits})$ and estimate which channel is used using these conditional probabilities. (5 p)

Hint: The estimate will be a so called *Bayes*-estimate.