Avd. Matematisk statistik

KTH Matematik

TENTAMEN I 5B1507 MATEMATISK STATISTIK FÖR T och M TORSDAGEN DEN 12 JANUARI 2006 KL 08.00–13.00.

Examinator: Jan Enger, tel. 790 7134.

Tillåtna hjälpmedel: Formel- och tabellsamling i Matematisk statistik. Formelblad statistisk kvalitetsstyrning. Räknare.

Införda beteckningar skall förklaras och definieras. Resonemang och uträkningar skall vara så utförliga och väl motiverade att de är lätta att följa. Numeriska svar skall anges med minst två siffrors noggrannhet. Varje korrekt lösning ger 10 poäng. Gränsen för godkänt är preliminärt 24 poäng.

Den som fått 22 eller 23 poäng på tentamen har möjlighet till komplettering. Kompletteringen skall göras inom två veckor efter det att resultatet av denna tentamen publicerats. Den som är aktuell för komplettering skall till examinator anmäla önskan att få en sådan inom en vecka från publicering av tentamensresultatet.

Den som fått godkänt på lappskrivning nummer 1 från den 23 september 2005, får uppgift 1 a) tillgodoräknad. Den som fått godkänt på lappskrivning nummer 2 från den 5 oktober 2005, får uppgift 4 a) tillgodoräknad. Tillgodoräknade uppgifter skall inte lösas.

Resultatet anslås senast torsdagen den 2 februari 2006 på Matematisk statistiks anslagstavla i entréplanet, Lindstedtsvägen 25, rakt fram innanför porten.

Tentamen kommer att finnas tillgänglig på elevexpeditionen sju veckor efter skrivningstillfället.

Uppgift 1

a) A discrete random variable X can take the values 1, 2, 3, 4 and 5 and has the probability function

k	1	2	3	4	5
$p_X(k)$	0.1	0.2	0.2	0.3	?

Calculate $p_X(5)$, the expected value $E(2X-3)$ and the standard deviation $D(2X-3)$. (5 p)

b) In an industrial process units are defective or correct independently of each other with probability p. If two units in a row from five consequately manufactured units are defective, the process is adjusted.

What is the probability that the process is adjusted for the first time when the seventh unit has been manufactured. Calculate also the numerical value for this probability when $p = 0.1$. (5 p)

Uppgift 2

a) Let X be chosen at random in the interval $(0\,,3)$ and assume that Y is, independently from X, chosen at random in the interval $(0\,,4)$, i.e. $X \in U(0,3)$ and $Y \in U(0,4)$.
Calculate $P(X \geq Y)$. (5 p)

b) Assume that $X_1, X_2, \ldots, X_{25}$ all are $U(0,3)$ and $Y_1, Y_2, \ldots, Y_{25}$ all are $U(0,4)$. All variables are assumed to be independent.
Calculate $P(\sum_{i=1}^{25} X_i \geq \sum_{i=1}^{25} Y_i)$. Well motivated approximations are allowed. (5 p)

Uppgift 3

In a production process the diameter of manufactured balls varies as $N(25.000, 0.020)$ mm. A ball whose diameter is outside the interval (24.930, 25.070) is rejected.

a) What is the probability that a manufactured ball is rejected? (4 p)

b) Out of 10000 manufactured balls, what is the probability that at most 5 are rejected. Well motivated approximations are allowed. (6 p)

Uppgift 4

a) A voltage meter has a systematic error 0.5 mV and a random error which is $N(0, 0.03)$ mV, and therefore a measurement of a voltage of μ mV gives a value which is $N(\mu{+}0.5, 0.03)$. In 6 measurement of an unknown voltage the following results were obtained (in mV)

10.53, 10.55, 10.55, 10.56, 10.57 and 10.57

Give a 95 % confidence interval for the unknown voltage, and test on the significance level 5% the hypothesis that the unknown voltage is 10.00. (5 p)

b) With another instrument the same voltage was measured and the following results were obtained.

9.99, 10.04, 10.04 och 10.05.

Independent, normally distributed observations with the same standard deviation as for the instrument in a).

Test, using the two samples, the hypothesis that the second instrument has no systematic error (i.e. that it is 0). Use 5 % significance level. (5 p)

Uppgift 5

a) The attenuation of the laser strength for a laser instrument, which is used for measurements of distances was measured for the distances 600, 800 and 1000 meter with the following results (where also the means for the attenuations for the different distances have been given).

Distance (x)	Observations (y)					mean value
600	10.1	12.1	10.7	11.5		11.1
800	15.3	16.7	15.5	17.1	16.9	16.3
1000	20.5	22.6	21.3	22.0		21.6

Numerical aid: $\sum_i (y_i - \bar{y})^2 = 228.0877, \quad \bar{y} = 16.3308$

a) Use a linear regression model $Y = \alpha + \beta x + \varepsilon,\ \varepsilon \in N(0, \sigma)$, and estimate the parameters α, β och σ^2 in it. (5 p)

b) Give a 95 % confidence interval for the expected attenuation at the distance 900 meter. (3 p)

c) Give a 95 % confidence interval for the slope of the regression line. (2 p)

Uppgift 6

To estimate the breaking effect of a car model the breaking distances were measured after breaking at the speeds 20, 40, 60, 80 and 100 km/h. Results:

speed v (20-s km/h)	1	2	3	4	5
breaking distance y (meter)	12	25	52	78	119

a) According to theory the breaking distance should follow a quadratic function of the speed plus a normally distributed error, and therefore the breaking distance breaking from the speed v is assumed to be N$(\gamma v^2, \sigma^2)$ where γ is unknown. Estimate γ with the Least square method. (4 p)

b) Check if the Least square estimate is unbiased. (3 p)

c) Calculate the variance for the Least square estimate in a) (in units of σ). (3 p)