Selected Exercises

(a) Probability

P1. The probability that a letter lies somewhere in a desk with seven drawers is p. It is equally likely to be in any one of the drawers. If six of them have been inspected without the letter having been found, find the probability that the letter lies in the seventh drawer.

P2. Find the mean and variance of $X \sim \chi^2(f)$.

P3. Let X_1, X_2, X_3, X_4, Y_1, Y_2, Y_3, Y_4 be independent rv's such that $X_i \sim N(4, 1)$, $i = 1, 2, 3, 4$ and $Y_i \sim N(3, 4)$, $i = 1, 2, 3, 4$. Determine

$$P(X_1 + 2X_2 + 3X_3 + 4X_4 < Y_1 + 2Y_2 + 3Y_3 + 4Y_4).$$

P4. (a) Throw two fair dice once and let X be the sum of the points. Find the distribution of X.
(b) Mark two fair dice with 1, 2, 2, 3, 3, 4 and 1, 3, 4, 5, 6, 8, respectively. Throw them once and let Y be the sum of the points. Find the distribution of Y.

P5. "A chain is no stronger than its weakest link." The load X when a link breaks is assumed to be Exp(50) (unit: newton). How many links can a chain at most consist of, if the probability that the chain breaks when the load is 10 newton must be at most 0.5?

P6. The two-dimensional rv (X, Y) has density function

$$f_{X,Y}(x, y) = \begin{cases} c \cdot x^2 y & 0 \le y \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Determine c.
(b) Determine the marginal densities.

P7. A factory produces 25 chairs of a certain type during a working-day. The probability is 0.95 that a chair is nondefective. Each chair is inspected before it is delivered, and defective chairs are later sold at a lower price. Find, approximately, the probability that more than 25 working-days are required to produce 600 nondefective chairs.

P8. Select two cards at random without replacement from an ordinary deck of cards. Find the probability that at least one of the cards is a heart.

P9. The two-dimensional rv (X, Y) has density function

$$f_{X,Y}(x, y) = c(y - x)^{\alpha}, \qquad 0 \le x < y < 1,$$

where $\alpha > -1$, $c = (\alpha + 1)(\alpha + 2)$. Find the means of X and Y.

P10. A box contains two defective and three nondefective transistors. To find the latter, the transistors are tested, one at a time, until either the two defective ones or the three nondefective ones have been found. Find the probability function of the required number, X, of tests.

P11. A test used to diagnose a certain disease has the following properties. It is positive with probability 0.99 if a person has the disease, and with probability 0.05 if a person does not have the disease. In the whole population, 1% has the disease. Determine the conditional probability that a person with a positive test has the disease.

P12. Let U_1 and U_2 be two urns containing in all two red balls and one white ball. Initially the balls are in U_1. First, one ball is drawn at random from U_1 and placed in U_2. Second, another drawing of one of the three balls is made at random; this ball is taken from the urn it is in and placed in the other urn. This procedure is repeated and stops as soon as a red ball is placed in U_2. Determine the probability that the white ball also then lies in U_2.

P13. A fair die is thrown until the same side turns up twice in succession. Determine the probability that this happens in throws $k - 1$ and k.

P14. In a food factory marmalade is filled into cardboard containers each of which weighs exactly 30 grams. The container is placed on a scale and filled with marmalade until the scale shows the weight m. The container then contains a total of Z grams of marmalade. The scale is subject to a random error $X \sim N(0, 7.5^2)$.
(a) Find the relation between Z, X and m.
(b) Find the distribution of Z.
(c) Choose m so that 95% of all containers hold at least 450 grams of marmalade.

P15. (Continuation of Exercise P14.) Assume that the weight of an empty container is no longer constant, but a rv $Y \sim N(30, 3^2)$. Assume also that X and Y are independent.
(a) Find the distribution of Z.
(b) How should m now be chosen?

P16. The rv X is Po(m). Determine, approximately, the mean and standard deviation of $Y = \sqrt{X}$.
Hint: Use Gauss's approximation formulae.

P17. The two-dimensional rv (X, Y) has density function

$$f_{X,Y}(x, y) = \tfrac{1}{2} \qquad \text{for} \quad |x| + |y| \le 1.$$

(a) Are X and Y uncorrelated?
(b) Are X and Y independent?

P18. The rv X has a Rayleigh distribution with density function $f_X(x) = 2\beta x e^{-\beta x^2}$, $x \ge 0$. Find $E(X)$ and $V(X)$.

P19. A fair coin is tossed three times. Let X be the number of heads at the first two throws and Y the total number of heads. Find the joint probability function of X and Y and the marginal probability functions.

P20. At a hamburger shop, three kinds of hamburgers are sold. The price (in English pounds (£)) of a hamburger is assumed to be a rv X which takes on the values 1, 1.20, 1.50 with probability 0.3, 0.2, 0.5. One day 300 hamburgers were sold. Find approximately the probability that the total sale that day amounted to at least £400.

P21. The weight, X, of a certain product is $N(10, 3)$. The products are divided into four classes as follows:

weight	class
$X \le 8$	A
$8 < X \le 9$	B
$9 < X \le 11$	C
$X > 11$	D

Compute the probability that, out of 10 randomly chosen items, 1, 3, 4 and 2 are in class A, B, C and D, respectively.

P22. The rv's X and Y are independent and uniformly distributed over the interval (0, 1). Determine the density function of the rv $Z = (X + Y)/2$.

P23. A bar of unit length is divided at random into two parts. Find the mean length of the smaller part.

P24. The two-dimensional rv (X, Y) has density function

$$f_{X,Y}(x, y) = 120xy(1 - x - y), \qquad x \ge 0, \quad y \ge 0, \quad x + y \le 1.$$

Find $C(X, Y)$.

P25. The rv's X and Y are independent and $\text{Bin}(n_1, p)$ and $\text{Bin}(n_2, p)$. Set $Z = \arcsin\sqrt{X/n_1} - \arcsin\sqrt{Y/n_2}$. Find approximately the mean and variance of Z.
Hint: Use Gauss's approximation formulae.

P26. A and B play the following game. A throws a fair die. If one or six turns up, he wins. Otherwise B throws the die repeatedly until either one of two events occurs: E = "B throws a one or a six" or F = "B throws the same outcome as A". B wins if E occurs and A wins if F occurs. Find the probability that A wins the game.

P27. A and B have bought ten fruits. Neither knows that three of the fruits are poisonous. A eats four fruits and B six. Find the probability that:
(a) A becomes poisoned;
(b) B becomes poisoned;
(c) both A and B become poisoned.

P28. The rv's X and Y are independent and uniformly distributed over the interval (0, 1). Derive the probability that either one of the variables is at least twice as large as the other.

P29. The triangle RST is equilateral with side 1. A point Q is chosen at random in the triangle. Derive the distribution function for the distance from Q to the side ST.

P30. A person buys three ropes of equal length. The strengths (unit: newton) of the ropes are independent with the same density function $10^{-6}xe^{-x/1000}$ $(x \geq 0)$.
(a) Find the probability that each rope breaks if the load is 1,500 newton.
(b) If the three ropes are tied into one long rope, what is the probability that it sustains a load of 1,500 newton without breaking?

P31. A person goes from one place to another, first by bus, then by taxi. The respective waiting-times (unit: minutes) are X and Y. The rv X has a uniform distribution in the interval (0, 10) and the rv Y has an exponential distribution with density function $(1/8)e^{-y/8}$, $y \geq 0$. The rv's X and Y are independent. Find the probability that the total waiting-time exceeds 16 minutes.

P32. A fair die is thrown n times. Find the probability that each side turns up at least once if:
(a) $n = 6$;
(b) $n = 7$.

P33. From an ordinary deck of cards 13 cards are drawn at random without replacement. Consider the events "six hearts occur" and "six diamonds occur". Determine the probability that at least one of these two events occurs.

P34. A person has two coins A and B. The coin A is an ordinary fair coin, and B is a false coin with heads on both sides. The person selects one coin at random.
(a) When the selected coin is tossed, it shows head. Find the probability that it is coin A.
(b) The selected coin is tossed once more, and again shows head. Find the probability that it is coin A.

P35. The rv's $X_1, X_2, \ldots, X_{100}$ are independent and uniformly distributed in the interval (0, 1). Let $Y = X_1 X_2 \cdot \ldots \cdot X_{100}$. Determine approximately the probability $P(Y < 10^{-40})$.

P36. A coin is such that $P(\text{head}) = p$, $P(\text{tail}) = q = 1 - p$. The coin is tossed until head has turned up r times. Let X be the number of trials performed.
(a) Find the probability function of X.
(b) Find the mean and variance of X.
Hint: In (b) you may if you like use the fact that $X = U_1 + \cdots + U_r$, where U_1 is the number of trials until the first head appears, and so on.

P37. The r-dimensional rv $(X_1, X_2, \ldots, X_r)$ has a multinomial distribution. (The usual symbols are used.) Show that

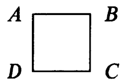
$$C(X_i, X_j) = -np_ip_j.$$

Hint: Consider $V(X_i + X_j)$.

P38. The rv's X_1 and X_2 are independent and $N(0, 1)$. Determine the density function of the rv $Y = (X_1^2 + X_2^2)^{1/2}$.

P39. Select n points at random on the circumference of a circle. Find the probability that all points lie on some semicircle.
Hint: Let $A_1, A_2, \ldots, A_n$ be the points counted in, say, the positive direction. To find the required probability, it suffices to consider each semicircle C_i beginning in A_i, $i = 1, 2, \ldots, n$.

P40. A person starts from A in the figure below and moves each time between the corners by going to one of the three other corners, selected at random with the same probability 1/3. Determine the mean number of moves until he has visited B, C and D.



(b) Statistics

S1. A very large batch of units has a relative frequency p of defectives where p is unknown but ≤ 0.04. It is required to take a random sample of n units from the batch and use this sample to construct an estimate of p with standard error of at most 0.02. How large should n be?

S2. Let $x_1, \ldots, x_n$ be independent observations on a rv with density function

$$(1/a)x^{(1/a)-1} \qquad (0 < x < 1; a > 0),$$

where the parameter a is unknown. Find the *ML* estimate of a and show that it is unbiased.

S3. Let

11.3 2.1 1.1 8.9 4.6 5.7 13.5 24.5 16.4

be independent observations on a rv X such that $\ln X \sim N(m, \sigma^2)$, where m and σ are unknown parameters. Construct a two-sided confidence interval with confidence level 95% for:
(a) m;
(b) σ.

S4. The data set $x_1, \ldots, x_{20}$ has arithmetic mean 2.0 and standard deviation 0.36. The data are transformed to new values $y_1, \ldots, y_{20}$ by the transformation $y_i = 1.0 + 0.4x_i$, $i = 1, \ldots, 20$. Compute the coefficient of variation of $y_1, \ldots, y_{20}$.

S5. The lifetime of certain components has density function

$$x^2e^{-x/a}/2a^3 \qquad (x > 0),$$

where a is an unknown parameter. The lifetimes of a random sample of n components are $x_1, \ldots, x_n$. Find the ML estimate of a and prove that it is unbiased.

S6. A method of measurement was studied by measuring two distances A and B, 10 times and 15 times, respectively. Results:

$$A: \sum_{1}^{10} x_i = 1{,}216; \qquad \sum_{1}^{10} x_i^2 = 168{,}731,$$

$$B: \sum_{1}^{15} y_i = 1{,}731; \qquad \sum_{1}^{15} y_i^2 = 232{,}654.$$

All errors of measurement are independent and $N(0, \sigma^2)$, where σ is unknown. Construct a two-sided confidence interval for σ with confidence level 0.95.

S7. An urn contains 7 balls, a of which are red, and the others blue. In order to test H_0: $a \leq 2$ against H_1: $a > 2$, two balls are drawn at random without replacement. The null hypothesis H_0 is rejected if both balls are red. Find the power of the test for all possible values of a.

S8. Let

1.21	0.20	0.39	0.68	2.00	1.07	0.96	0.55	0.80
2.45	0.67	0.34	0.22	0.77	1.24	0.39	0.29	0.59

be a random sample of 18 values from a distribution with density function $\exp[-(x-a)]$, $x > a$, where a is unknown. Test the hypothesis $a = 0$ against the hypothesis $a > 0$ at the level of significance 5%, using the smallest value in the sample as a test quantity.

S9. Consider a regression model with n pairs (x_i, y_i) where y_i is an observation on $Y_i \sim N(\beta x_i, 1)$ and β is unknown. Find the ML estimate of β and its variance.

S10. The rv Y has a geometric distribution such that $P(Y = k) = p(1-p)^k$, $k = 0, 1, \ldots$. The parameter p is unknown. We have the following random sample of 50 values from this distribution:

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1 0 1 0 1

0 1 0 0 2 1 1 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0.

Find:
(a) the ML estimate of p;
(b) the LS estimate of p.

S11. Let

1.8807	0.1251	2.0899	2.1059	1.8722	0.1346	2.2767	1.5771
−0.7184	−0.2791	2.1957	2.2535	3.0155	0.9356	−0.4501	0.7535
2.0769	1.1986	0.3574	2.5372	2.6740	1.3706	1.0940	2.8718
0.2142	2.0177	−0.5822	0.7159	2.7182	0.9693		

be a random sample of 30 observations from $N(m, m)$, where m is unknown. Find the ML estimate of m.

S12. Let

12 0 6 2 9 and 14 11 7 12 16

be random samples from $\text{Po}(m_1)$ and $\text{Po}(m_2)$, respectively. Test the hypothesis $m_1 = m_2$ with an approximate two-sided test.

S13. A person counts the number of trucks passing his bedroom window from 11 p.m. to 7 a.m. during three nights. The number passing during one night is supposed to be $\text{Po}(\lambda)$, where λ is unknown. He obtained the numbers 136, 154 and 127.

(a) Prove that the ML estimate and the LS estimate of λ are identical.

(b) Construct a two-sided confidence interval for λ with approximate confidence level 99%.

S14. In a factory the method of production has been changed. In order to find out whether the quality of a certain product has been affected, a random sample of 100 units produced before the change was compared with a similar sample produced after the change. It appeared that 36 units produced before the change and 27 units produced after the change did not meet specifications. Test the hypothesis H_0 that the quality has not been affected.

S15. Let

3.24 3.37 3.29 3.18 3.51

3.20 3.35 3.38 3.29 3.42

be a random sample from $N(\ln(M + 1), \sigma^2)$ where M and σ are unknown. Construct a two-sided confidence interval for M with confidence level 0.95.

S16. Let $x_1, \ldots, x_{15}$ be a random sample from $\text{Exp}(a)$, where a is unknown. It is found that $\sum x_i = 27$. Test H_0: $a = 1$ against H_1: $a > 1$ with an exact test at the level of significance 0.05.

S17. Let

2 10 6 3 6 3

be a random sample from $\text{Po}(4\lambda)$, where λ is unknown. Construct a one-sided confidence interval $(0, a)$ for λ with the approximate confidence level 99%.

S18. When analysing n samples according to two methods A and B, a chemist obtained n pairs of values (x_i, y_i), $i = 1, \ldots, n$. Model: x_i is an observation on $X_i \sim N(m_i, \sigma_1^2)$ and y_i is an observation on $Y_i \sim N(m_i, \sigma_2^2)$. The parameter σ_1 is known, but the parameters $m_1, \ldots, m_n$ and σ_2 are unknown. Describe how a two-sided 95% confidence interval for σ_2 is found.

S19. The rv X assumes the values 1, 2 and 3 with probabilities θ, θ and $1 - 2\theta$, where θ is an unknown parameter. Let $x_1, \ldots, x_n$ be a random sample from this distribution.

(a) Derive the ML estimate of θ.

(b) Describe how the ML estimate can be used for testing hypotheses concerning θ both when n is small and when n is large.

S20. A chemist wants to compare two pH meters, called type A and type B. He believes that there exists a systematic difference d, such that type B delivers, on the average, smaller values than type A. Therefore the pH value was determined for five different solutions, using both instruments. Results:

Solution	1	2	3	4	5
Type A	6.23	4.16	8.79	10.11	3.56
Type B	6.10	3.96	8.82	9.83	3.50

Test the chemist's belief, using a suitable test of significance at the level of significance 5%. Normal distributions may be assumed.

S21. Let $x_1, \ldots, x_n$ be independent observations on a rv which has a uniform distribution over the interval $(-a, a)$. The constant a is unknown. Show that $c\sum_1^n x_i^2$ is an unbiased estimate of the parameter a^2 if c is suitably chosen. Prove that the estimate is consistent.

S22. An object consists of two parts A and B which have been weighed three and four times, respectively. Further, the whole object has been weighed twice, using the same balance. Results:

A	12.07	12.01	12.04	
B	18.34	18.36	18.35	18.32
$A + B$	30.35	30.39		

Each result is subject to a random error with mean 0 and standard deviation σ. Find the LS estimate of the total weight of the object.

S23. Let $x_1, \ldots, x_n$ be a random sample from $N(m, \sigma^2)$ where m is known and $\theta = \sigma^2$ is unknown. The estimates

$$\theta_1^* = \sum_1^n (x_i - m)^2/n,$$

$$\theta_2^* = \sum_1^n (x_i - \bar{x})^2/(n - 1),$$

are both unbiased. Which of them has the smallest variance?

S24. A machine produces units with weights (in grams) distributed as $N(50.0, \sigma^2)$, where σ is unknown. A unit is considered to be nondefective if its weight is between 47.0 and 53.0. To control σ, units are weighed, one at a time, until a defective unit is obtained.

On one occasion, the first defective unit was found when the tenth unit was weighed. Determine the ML estimate of σ.

S25. A firm asserts that a certain measuring device is such that the absolute error (that is, the absolute value of the difference between the measurement and the correct value) exceeds 0.05 in at most one case in 10. A person assumes that the error is $N(0, \sigma^2)$ but does not believe σ to be so small as to make the assertion correct. He therefore performs 15 independent measurements and obtains the errors

−0.027	0.012	0.039	−0.062	−0.032	0.074	−0.006	0.013
−0.019	0.010	0.047	0.039	0.005	−0.048	0.016	

Test the assertion of the firm.

S26. Let $x_1, \ldots, x_{16}$ be a random sample from $N(m, 1)$. Reject the hypothesis H_0: $m = 0$ if $|\bar{x}| > 0.5$. Derive the power function $h(m)$ of the test and draw its graph.

S27. Consider the following data:

x_i	1	2	3	4	5	6
y_i	3.0	3.9	5.6	6.7	8.8	10.1

Model: y_i is an observation from $N(\alpha + \beta x_i, \sigma^2)$ where α, β and σ^2 are unknown parameters. Test the hypothesis H_0: $\alpha = 0$ with a two-sided test of significance.

S28. The numbers of dots (defects) in two types, A and B, of laminates of a certain size are assumed to be $\text{Po}(m_A)$ and $\text{Po}(m_B)$, respectively. On 30 laminates of type A and 45 of type B, the following numbers of dots were found:

A	1	3	1	0	0	0	2	1	1	0	2	0	0	2	0
	1	0	2	0	0	2	0	0	1	1	0	0	1	0	0
B	0	0	0	0	0	2	0	0	1	1	1	0	0	0	0
	0	1	0	0	1	0	0	1	0	1	0	0	0	0	0
	0	0	0	1	0	0	0	0	2	1	0	1	1	1	0

Estimate the difference $m_A - m_B$ and find a standard error of the difference.

S29. Let $x_1, \ldots, x_8$ and $y_1, \ldots, y_{12}$ be random samples from $\text{Exp}(\theta)$ and $\text{Exp}(3\theta)$, respectively, where θ is an unknown parameter. It is found that $\sum x_i = 19.0$, $\sum y_i = 63.6$. Derive the ML estimate of θ and find its variance.

S30. The maximum height H of the waves during one year at a certain place is assumed to be a rv having a Rayleigh distribution with density function

$$f_H(x) = (x/a)e^{-x^2/2a} \qquad (x > 0),$$

where a is an unknown parameter. During 8 years the following heights (in meters) have been observed:

2.5 2.9 1.8 0.9 1.7 2.1 2.2 2.8.

Use the ML estimate of a to find an estimate of the "1,000 year wave", that is, a wave so high that it occurs, on average, only once in a thousand years.

S31. A physicist has performed five measurements of a physical constant m. The measurements are assumed to be $N(m, \sigma^2)$ with known variance σ^2. The phys-

icist obtained the 90% confidence interval (7.02, 7.14). How many additional measurements are required if the physicist wants:
(a) a confidence interval with half the length;
(b) a confidence interval with the same length but 95% confidence level?

S32. A person has an observation x on $X \sim \text{Bin}(10, p)$, where p is unknown. He used the normal approximation to construct the confidence interval

$$I_p = (x/10 \pm 1.96\sqrt{x(10 - x)/1{,}000}).$$

The desired confidence level was 95%. The true confidence level is a function $K(p)$ of p. Find $K(0.50)$.

S33. A market research institute wants to estimate the fraction p in a population of 1,200 families who would be willing to buy a certain product. The institute therefore plans to ask n persons, drawn at random without replacement, if they want or do not want to buy the product. It is known beforehand that at most 1/4 will be positive. Find n so that a confidence interval for p with approximate confidence level 95% and total length 0.05 can be constructed.

S34. Let $x_1, \ldots, x_9$ be a random sample from $N(m, 1)$. Construct a test of H_0: $m = 2$ against H_1: $m \neq 2$ at the level of significance 5%.
(a) Derive the power function of the test.
(b) Suppose that the power is not good enough. How many values are needed in order to give the test a power of 0.99 for $m = 1$?

S35. A dentist wants to compare two anaesthetics A and B by testing them on each of 15 patients. He is interested to know whether an injection results in anaesthesia (event H) or does not result in any anaesthesia (event H^*). Results:

	Patient no.															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	H	H	H	H	H	H^*	H	H	H	H	H	H	H^*	H	H	H
B	H	H	H^*	H	H^*	H^*	H^*	H^*	H	H	H^*	H^*	H	H	H^*	H^*

Test whether A and B differ with respect to the incidence of anaesthesia. Determine the P value (two-sided test).

S36. A pond contains N fish, where N is unknown. In order to estimate N, one catches 100 fish, marks them and returns them to the pond. A day later, 400 fish are caught, 42 of which are marked. Construct an approximate two-sided 95% confidence interval for N, under suitable assumptions.

S37. When studying the velocity of a chemical reaction at constant temperature, a chemist determined at different times t_i the concentration y_i of a substance participating in the reaction. Results:

t_i	0	184	319	400	526	575
y_i	2.33	2.08	1.91	1.82	1.67	1.62

As a model the chemist chose $y_i = c \cdot \exp(\beta t_i) \cdot \varepsilon_i$ where $\ln \varepsilon_i$ is an observation from $N(0, \sigma^2)$ and the parameters c, β and σ are unknown. Determine a 95% confidence interval for $m = c \cdot \exp(600\beta)$.

S38. A product is manufactured in units. If the length of a unit exceeds 10 (mm), it is defective. The lengths are assumed to be $N(m, 0.1^2)$. In order to control a large batch, the manufacturer selects n units and will accept the batch if and only if the arithmetic mean of the lengths of these units is less than a.

Find a and n so that the following conditions are fulfilled:

(a) The probability is 90% that a batch with 0.1% defective units is accepted.

(b) The probability is 5% that a batch with 1% defective units is accepted.

S39. Consider the usual regression model with n pairs (x_i, y_i) where y_i is an observation on $Y_i \sim N(\alpha + \beta(x_i - \bar{x}), \sigma^2)$. The parameters α, β and σ are unknown. Show that the ML estimates of α and β coincide with the LS estimates.

S40. A circle with known centre has the unknown radius R. In the circle n points are chosen at random. The distances from the points to the centre are $x_1, \ldots, x_n$. Find an unbiased estimate of R based on $\max(x_i)$.

References

This list of references is longer than is usual in elementary books. It is intended to demonstrate the breadth of probability and statistics as reflected in the literature. Therefore, theoretical and applied areas not treated in the present book are also represented in the list. Most books are on an elementary or intermediate level, but there are also some advanced titles.

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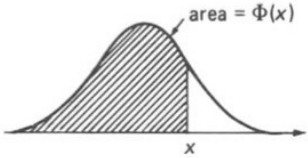
Tables[1]

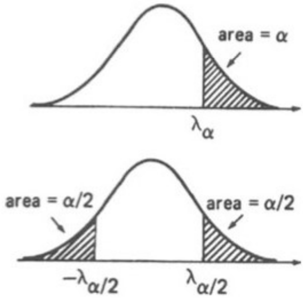
[1] The tables are taken from a Swedish publication (see Preface), and the same numbering of the tables has been used as in that publication.

Table 1. Normal distribution. $\Phi(x) = P(X \leq x)$ where $X \sim N(0, 1)$. When x is negative, use the fact that $\Phi(-x) = 1 - \Phi(x)$.



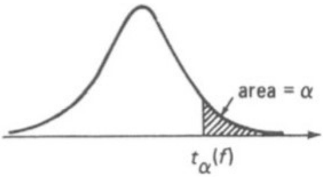
x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865									
3.1	.99903									
3.2	.99931									
3.3	.99952									
3.4	.99966									
3.5	.99977									
3.6	.99984									
3.7	.99989									
3.8	.99993									
3.9	.99995									
4.0	.99997									

Table 2. Quantiles of normal distribution. $P(X > \lambda_\alpha) = \alpha$ where $X \sim N(0, 1)$.



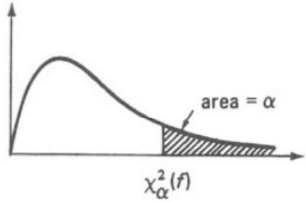
α	λ_α	α	λ_α
0.10	1.2816	0.001	3.0902
0.05	1.6449	0.0005	3.2905
0.025	1.9600	0.0001	3.7190
0.010	2.3263	0.00005	3.8906
0.005	2.5758	0.00001	4.2649

Table 3. t Distribution. $P(X > t_\alpha(f)) = \alpha$ where $X \sim t(f)$.



f \ α	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.08	6.31	12.71	31.82	63.66	318.31	636.61
2	1.89	2.92	4.30	6.96	9.92	22.33	31.60
3	1.64	2.35	3.18	4.54	5.84	10.21	12.92
4	1.53	2.13	2.78	3.75	4.60	7.17	8.61
5	1.48	2.02	2.57	3.36	4.03	5.89	6.87
6	1.44	1.94	2.45	3.14	3.71	5.21	5.96
7	1.41	1.89	2.36	3.00	3.50	4.79	5.41
8	1.40	1.86	2.31	2.90	3.36	4.50	5.04
9	1.38	1.83	2.26	2.82	3.25	4.30	4.78
10	1.37	1.81	2.23	2.76	3.17	4.14	4.59
11	1.36	1.80	2.20	2.72	3.11	4.02	4.44
12	1.36	1.78	2.18	2.68	3.05	3.93	4.32
13	1.35	1.77	2.16	2.65	3.01	3.85	4.22
14	1.34	1.76	2.14	2.62	2.98	3.79	4.14
15	1.34	1.75	2.13	2.60	2.95	3.73	4.07
16	1.34	1.75	2.12	2.58	2.92	3.69	4.02
17	1.33	1.74	2.11	2.57	2.90	3.65	3.97
18	1.33	1.73	2.10	2.55	2.88	3.61	3.92
19	1.33	1.73	2.09	2.54	2.86	3.58	3.88
20	1.33	1.72	2.09	2.53	2.85	3.55	3.85
21	1.32	1.72	2.08	2.52	2.83	3.53	3.82
22	1.32	1.72	2.07	2.51	2.82	3.51	3.79
23	1.32	1.71	2.07	2.50	2.81	3.48	3.77
24	1.32	1.71	2.06	2.49	2.80	3.47	3.75
25	1.32	1.71	2.06	2.49	2.79	3.45	3.73
26	1.32	1.71	2.06	2.48	2.78	3.44	3.71
27	1.31	1.70	2.05	2.47	2.77	3.42	3.69
28	1.31	1.70	2.05	2.47	2.76	3.41	3.67
29	1.31	1.70	2.05	2.46	2.76	3.40	3.66
30	1.31	1.70	2.04	2.46	2.75	3.39	3.65
40	1.30	1.68	2.02	2.42	2.70	3.31	3.55
60	1.30	1.67	2.00	2.39	2.66	3.23	3.46
120	1.29	1.66	1.98	2.36	2.62	3.16	3.37
∞	1.28	1.64	1.96	2.33	2.58	3.09	3.29

Table 4. χ^2 Distribution. $P(X > \chi^2_\alpha(f)) = \alpha$ where $X \sim \chi^2(f)$.



f \ α	0.9995	0.999	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.001	0.0005
1	0.00	0.00	0.00	0.00	0.00	0.00	3.84	5.02	6.63	7.88	10.8	12.1
2	0.00	0.00	0.01	0.02	0.05	0.10	5.99	7.38	9.21	10.6	13.8	15.2
3	0.02	0.02	0.07	0.12	0.22	0.35	7.81	9.35	11.3	12.8	16.3	17.7
4	0.06	0.09	0.21	0.30	0.48	0.71	9.49	11.1	13.3	14.9	18.5	20.0
5	0.16	0.21	0.41	0.55	0.83	1.15	11.1	12.8	15.1	16.7	20.5	22.1
6	0.30	0.38	0.68	0.87	1.24	1.64	12.6	14.4	16.8	18.5	22.5	24.1
7	0.48	0.60	0.99	1.24	1.69	2.17	14.1	16.0	18.5	20.3	24.3	26.0
8	0.71	0.86	1.34	1.65	2.18	2.73	15.5	17.5	20.1	22.0	26.1	27.9
9	0.97	1.15	1.73	2.09	2.70	3.33	16.9	19.0	21.7	23.6	27.9	29.7
10	1.26	1.48	2.16	2.56	3.25	3.94	18.3	20.5	23.2	25.2	29.6	31.4
11	1.59	1.83	2.60	3.05	3.82	4.57	19.7	21.9	24.7	26.8	31.3	33.1
12	1.93	2.21	3.07	3.57	4.40	5.23	21.0	23.3	26.2	28.3	32.9	34.8
13	2.31	2.62	3.57	4.11	5.01	5.89	22.4	24.7	27.7	29.8	34.5	36.5
14	2.70	3.04	4.07	4.66	5.63	6.57	23.7	26.1	29.1	31.3	36.1	38.1
15	3.11	3.48	4.60	5.23	6.26	7.26	25.0	27.5	30.6	32.8	37.7	39.7
16	3.54	3.94	5.14	5.81	6.91	7.96	26.3	28.8	32.0	34.3	39.3	41.3
17	3.98	4.42	5.70	6.41	7.56	8.67	27.6	30.2	33.4	35.7	40.8	42.9
18	4.44	4.90	6.26	7.01	8.23	9.39	28.9	31.5	34.8	37.2	42.3	44.4
19	4.91	5.41	6.84	7.63	8.91	10.1	30.1	32.9	36.2	38.6	43.8	46.0
20	5.40	5.92	7.43	8.26	9.59	10.9	31.4	34.2	37.6	40.0	45.3	47.5
21	5.90	6.45	8.03	8.90	10.3	11.6	32.7	35.5	38.9	41.4	46.8	49.0
22	6.40	6.98	8.64	9.54	11.0	12.3	33.9	36.8	40.3	42.8	48.3	50.5
23	6.92	7.53	9.26	10.2	11.7	13.1	35.2	38.1	41.6	44.2	49.7	52.0
24	7.45	8.08	9.89	10.9	12.4	13.8	36.4	39.4	43.0	45.6	51.2	53.5
25	7.99	8.65	10.5	11.5	13.1	14.6	37.7	40.6	44.3	46.9	52.6	54.9
26	8.54	9.22	11.2	12.2	13.8	15.4	38.9	41.9	45.6	48.3	54.1	56.4
27	9.09	9.80	11.8	12.9	14.6	16.2	40.1	43.2	47.0	49.6	55.5	57.9
28	9.66	10.4	12.5	13.6	15.3	16.9	41.3	44.5	48.3	51.0	56.9	59.3
29	10.2	11.0	13.1	14.3	16.0	17.7	42.6	45.7	49.6	52.3	58.3	60.7
30	10.8	11.6	13.8	15.0	16.8	18.5	43.8	47.0	50.9	53.7	59.7	62.2
40	16.9	17.9	20.7	22.2	24.4	26.5	55.8	59.3	63.7	66.8	73.4	76.1
50	23.5	24.7	28.0	29.7	32.4	34.8	67.5	71.4	76.2	79.5	86.7	89.6
60	30.3	31.7	35.5	37.5	40.5	43.2	79.1	83.3	88.4	92.0	99.6	103.
70	37.5	39.0	43.3	45.4	48.8	51.7	90.5	95.0	100.	104.	112.	116.
80	44.8	46.5	51.2	53.5	57.2	60.4	102.	107.	112.	116.	125.	128.
90	52.3	54.2	59.2	61.8	65.6	69.1	113.	118.	124.	128.	137.	141.
100	59.9	61.9	67.3	70.1	74.2	77.9	124.	130.	136.	140.	149.	153.

Table 7. Poisson distribution. $P(X \leq x)$ where $X \sim \mathrm{Po}(m)$.

x \ m	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	.90484	.81873	.74082	.67032	.60653	.54881	.49659	.44933	.40657
1	.99532	.98248	.96306	.93845	.90980	.87810	.84420	.80879	.77248
2	.99985	.99885	.99640	.99207	.98561	.97688	.96586	.95258	.93714
3	1.00000	.99994	.99973	.99922	.99825	.99664	.99425	.99092	.98654
4		1.00000	.99998	.99994	.99983	.99961	.99921	.99859	.99766
5			1.00000	1.00000	.99999	.99996	.99991	.99982	.99966
6					1.00000	1.00000	.99999	.99998	.99996
7							1.00000	1.00000	1.00000

x \ m	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
0	.36788	.30119	.24660	.20190	.16530	.13534	.11080	.09072	.07427
1	.73576	.66263	.59183	.52493	.46284	.40601	.35457	.30844	.26738
2	.91970	.87949	.83350	.78336	.73062	.67668	.62271	.56971	.51843
3	.98101	.96623	.94627	.92119	.89129	.85712	.81935	.77872	.73600
4	.99634	.99225	.98575	.97632	.96359	.94735	.92750	.90413	.87742
5	.99941	.99850	.99680	.99396	.98962	.98344	.97509	.96433	.95096
6	.99992	.99975	.99938	.99866	.99743	.99547	.99254	.98841	.98283
7	.99999	.99996	.99989	.99974	.99944	.99890	.99802	.99666	.99467
8	1.00000	1.00000	.99998	.99995	.99989	.99976	.99953	.99914	.99851
9			1.00000	.99999	.99998	.99995	.99990	.99980	.99962
10				1.00000	1.00000	.99999	.99998	.99996	.99991
11						1.00000	1.00000	.99999	.99998
12								1.00000	1.00000

x \ m	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4
0	.06081	.04979	.04076	.03337	.02732	.02237	.01832	.01500	.01228
1	.23108	.19915	.17120	.14684	.12569	.10738	.09158	.07798	.06630
2	.46945	.42319	.37990	.33974	.30275	.26890	.23810	.21024	.18514
3	.69194	.64723	.60252	.55836	.51522	.47348	.43347	.39540	.35945
4	.84768	.81526	.78061	.74418	.70644	.66784	.62884	.58983	.55118
5	.93489	.91608	.89459	.87054	.84412	.81556	.78513	.75314	.71991
6	.97559	.96649	.95538	.94215	.92673	.90911	.88933	.86746	.84365
7	.99187	.98810	.98317	.97693	.96921	.95989	.94887	.93606	.92142
8	.99757	.99620	.99429	.99171	.98833	.98402	.97864	.97207	.96420
9	.99934	.99890	.99824	.99729	.99598	.99420	.99187	.98887	.98511
10	.99984	.99971	.99950	.99919	.99873	.99807	.99716	.99593	.99431
11	.99996	.99993	.99987	.99978	.99963	.99941	.99908	.99863	.99799
12	.99999	.99998	.99997	.99994	.99990	.99983	.99973	.99957	.99934
13	1.00000	1.00000	.99999	.99999	.99997	.99996	.99992	.99987	.99980
14			1.00000	1.00000	.99999	.99999	.99998	.99997	.99994
15					1.00000	1.00000	1.00000	.99999	.99998
16								1.00000	1.00000

Table 7 (*continued*)

x \ m	4.6	4.8	5.0	5.5	6.0	6.5	7.0	7.5	8.0
0	.01005	.00823	.00674	.00409	.00248	.00150	.00091	.00055	.00034
1	.05629	.04773	.04043	.02656	.01735	.01128	.00730	.00470	.00302
2	.16264	.14254	.12465	.08838	.06197	.04304	.02964	.02026	.01375
3	.32571	.29423	.26503	.20170	.15120	.11185	.08177	.05915	.04238
4	.51323	.47626	.44049	.35752	.28506	.22367	.17299	.13206	.09963
5	.68576	.65101	.61596	.52892	.44568	.36904	.30071	.24144	.19124
6	.81803	.79080	.76218	.68604	.60630	.52652	.44971	.37815	.31337
7	.90495	.88667	.86663	.80949	.74398	.67276	.59871	.52464	.45296
8	.95493	.94418	.93191	.89436	.84724	.79157	.72909	.66197	.59255
9	.98047	.97486	.96817	.94622	.91608	.87738	.83050	.77641	.71662
10	.99222	.98958	.98630	.97475	.95738	.93316	.90148	.86224	.81589
11	.99714	.99601	.99455	.98901	.97991	.96612	.94665	.92076	.88808
12	.99902	.99858	.99798	.99555	.99117	.98397	.97300	.95733	.93620
13	.99969	.99953	.99930	.99831	.99637	.99290	.98719	.97844	.96582
14	.99991	.99985	.99977	.99940	.99860	.99704	.99428	.98974	.98274
15	.99997	.99996	.99993	.99980	.99949	.99884	.99759	.99539	.99177
16	.99999	.99999	.99998	.99994	.99983	.99957	.99904	.99804	.99628
17	1.00000	1.00000	.99999	.99998	.99994	.99985	.99964	.99921	.99841
18			1.00000	.99999	.99998	.99995	.99987	.99970	.99935
19				1.00000	.99999	.99998	.99996	.99989	.99975
20					1.00000	1.00000	.99999	.99996	.99991
21							1.00000	.99999	.99997
22								1.00000	.99999
23									1.00000

Table 7 (*continued*)

x \ m	8.5	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0
0	.00020	.00012	.00007	.00005	.00002	.00001	.00000	.00000	.00000
1	.00193	.00123	.00079	.00050	.00020	.00008	.00003	.00001	.00000
2	.00928	.00623	.00416	.00277	.00121	.00052	.00022	.00009	.00004
3	.03011	.02123	.01486	.01034	.00492	.00229	.00105	.00047	.00021
4	.07436	.05496	.04026	.02925	.01510	.00760	.00374	.00181	.00086
5	.14960	.11569	.08853	.06709	.03752	.02034	.01073	.00553	.00279
6	.25618	.20678	.16495	.13014	.07861	.04582	.02589	.01423	.00763
7	.38560	.32390	.26866	.22022	.14319	.08950	.05403	.03162	.01800
8	.52311	.45565	.39182	.33282	.23199	.15503	.09976	.06206	.03745
9	.65297	.58741	.52183	.45793	.34051	.24239	.16581	.10940	.06985
10	.76336	.70599	.64533	.58304	.45989	.34723	.25168	.17568	.11846
11	.84866	.80301	.75199	.69678	.57927	.46160	.35316	.26004	.18475
12	.90908	.87577	.83643	.79156	.68870	.57597	.46310	.35846	.26761
13	.94859	.92615	.89814	.86446	.78129	.68154	.57304	.46445	.36322
14	.97257	.95853	.94001	.91654	.85404	.77202	.67513	.57044	.46565
15	.98617	.97796	.96653	.95126	.90740	.84442	.76361	.66936	.56809
16	.99339	.98889	.98227	.97296	.94408	.89871	.83549	.75592	.66412
17	.99700	.99468	.99107	.98572	.96781	.93703	.89046	.82720	.74886
18	.99870	.99757	.99572	.99281	.98231	.96258	.93017	.88264	.81947
19	.99947	.99894	.99804	.99655	.99071	.97872	.95733	.92350	.87522
20	.99979	.99956	.99914	.99841	.99533	.98840	.97499	.95209	.91703
21	.99992	.99983	.99964	.99930	.99775	.99393	.98592	.97116	.94689
22	.99997	.99993	.99985	.99970	.99896	.99695	.99238	.98329	.96726
23	.99999	.99998	.99994	.99988	.99954	.99853	.99603	.99067	.98054
24	1.00000	.99999	.99998	.99995	.99980	.99931	.99801	.99498	.98884
25		1.00000	.99999	.99998	.99992	.99969	.99903	.99739	.99382
26			1.00000	.99999	.99997	.99987	.99955	.99869	.99669
27				1.00000	.99999	.99994	.99980	.99936	.99828
28					1.00000	.99998	.99991	.99970	.99914
29					1.00000	.99999	.99996	.99986	.99958
30						1.00000	.99998	.99994	.99980
31						1.00000	.99999	.99997	.99991
32							1.00000	.99999	.99996
33								1.00000	.99998
34								1.00000	.99999
35									1.00000

Table 8. Binomial distribution. $P(X \leq x)$ where $X \sim \text{Bin}(n, p)$. When $p > 1/2$, use the fact that $P(X \leq x) = P(Y \geq n - x)$ where $Y \sim \text{Bin}(n, 1 - p)$.

n	x \ p	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
2	0	.90250	.81000	.72250	.64000	.56250	.49000	.36000	.25000
	1	.99750	.99000	.97750	.96000	.93750	.91000	.84000	.75000
3	0	.85738	.72900	.61413	.51200	.42188	.34300	.21600	.12500
	1	.99275	.97200	.93925	.89600	.84375	.78400	.64800	.50000
	2	.99988	.99900	.99663	.99200	.98438	.97300	.93600	.87500
4	0	.81451	.65610	.52201	.40960	.31641	.24010	.12960	.06250
	1	.98598	.94770	.89048	.81920	.73828	.65170	.47520	.31250
	2	.99952	.99630	.98802	.97280	.94922	.91630	.82080	.68750
	3	.99999	.99990	.99949	.99840	.99609	.99190	.97440	.93750
5	0	.77378	.59049	.44371	.32768	.23730	.16807	.07776	.03125
	1	.97741	.91854	.83521	.73728	.63281	.52822	.33696	.18750
	2	.99884	.99144	.97339	.94208	.89648	.83692	.68256	.50000
	3	.99997	.99954	.99777	.99328	.98438	.96922	.91296	.81250
	4	1.00000	.99999	.99992	.99968	.99902	.99757	.98976	.96875
6	0	.73509	.53144	.37715	.26214	.17798	.11765	.04666	.01563
	1	.96723	.88573	.77648	.65536	.53394	.42018	.23328	.10938
	2	.99777	.98415	.95266	.90112	.83057	.74431	.54432	.34375
	3	.99991	.99873	.99411	.98304	.96240	.92953	.82080	.65625
	4	1.00000	.99994	.99960	.99840	.99536	.98907	.95904	.89063
	5	1.00000	1.00000	.99999	.99994	.99976	.99927	.99590	.98438
7	0	.69834	.47830	.32058	.20972	.13348	.08235	.02799	.00781
	1	.95562	.85031	.71658	.57672	.44495	.32942	.15863	.06250
	2	.99624	.97431	.92623	.85197	.75641	.64707	.41990	.22656
	3	.99981	.99727	.98790	.96666	.92944	.87396	.71021	.50000
	4	.99999	.99982	.99878	.99533	.98712	.97120	.90374	.77344
	5	1.00000	.99999	.99993	.99963	.99866	.99621	.98116	.93750
	6	1.00000	1.00000	1.00000	.99999	.99994	.99978	.99836	.99219
8	0	.66342	.43047	.27249	.16777	.10011	.05765	.01680	.00391
	1	.94276	.81310	.65718	.50332	.36708	.25530	.10638	.03516
	2	.99421	.96191	.89479	.79692	.67854	.55177	.31539	.14453
	3	.99963	.99498	.97865	.94372	.88618	.80590	.59409	.36328
	4	.99998	.99957	.99715	.98959	.97270	.94203	.82633	.63672
	5	1.00000	.99998	.99976	.99877	.99577	.98871	.95019	.85547
	6	1.00000	1.00000	.99999	.99992	.99962	.99871	.99148	.96484
	7	1.00000	1.00000	1.00000	1.00000	.99998	.99993	.99934	.99609
9	0	.63025	.38742	.23162	.13422	.07508	.04035	.01008	.00195
	1	.92879	.77484	.59948	.43621	.30034	.19600	.07054	.01953
	2	.99164	.94703	.85915	.73820	.60068	.46283	.23179	.08984
	3	.99936	.99167	.96607	.91436	.83427	.72966	.48261	.25391
	4	.99997	.99911	.99437	.98042	.95107	.90119	.73343	.50000
	5	1.00000	.99994	.99937	.99693	.99001	.97471	.90065	.74609
	6	1.00000	1.00000	.99995	.99969	.99866	.99571	.97497	.91016
	7	1.00000	1.00000	1.00000	.99998	.99989	.99957	.99620	.98047
	8	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99974	.99805

Table 8 (*continued*)

n	x \ p	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
10	0	.59874	.34868	.19687	.10737	.05631	.02825	.00605	.00098
	1	.91386	.73610	.54430	.37581	.24403	.14931	.04636	.01074
	2	.98850	.92981	.82020	.67780	.52559	.38278	.16729	.05469
	3	.99897	.98720	.95003	.87913	.77588	.64961	.38228	.17188
	4	.99994	.99837	.99013	.96721	.92187	.84973	.63310	.37695
	5	1.00000	.99985	.99862	.99363	.98027	.95265	.83376	.62305
	6	1.00000	.99999	.99987	.99914	.99649	.98941	.94524	.82813
	7	1.00000	1.00000	.99999	.99992	.99958	.99841	.98771	.94531
	8	1.00000	1.00000	1.00000	1.00000	.99997	.99986	.99832	.98926
	9	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99990	.99902
11	0	.56880	.31381	.16734	.08590	.04224	.01977	.00363	.00049
	1	.89811	.69736	.49219	.32212	.19710	.11299	.03023	.00586
	2	.98476	.91044	.77881	.61740	.45520	.31274	.11892	.03271
	3	.99845	.98147	.93056	.83886	.71330	.56956	.29628	.11328
	4	.99989	.99725	.98411	.94959	.88537	.78970	.53277	.27441
	5	.99999	.99970	.99734	.98835	.96567	.92178	.75350	.50000
	6	1.00000	.99998	.99968	.99803	.99244	.97838	.90065	.72559
	7	1.00000	1.00000	.99997	.99976	.99881	.99571	.97072	.88672
	8	1.00000	1.00000	1.00000	.99998	.99987	.99942	.99408	.96729
	9	1.00000	1.00000	1.00000	1.00000	.99999	.99995	.99927	.99414
	10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99996	.99951
12	0	.54036	.28243	.14224	.06872	.03168	.01384	.00218	.00024
	1	.88164	.65900	.44346	.27488	.15838	.08503	.01959	.00317
	2	.98043	.88913	.73582	.55835	.39068	.25282	.08344	.01929
	3	.99776	.97436	.90779	.79457	.64878	.49252	.22534	.07300
	4	.99982	.99567	.97608	.92744	.84236	.72366	.43818	.19385
	5	.99999	.99946	.99536	.98059	.94560	.88215	.66521	.38721
	6	1.00000	.99995	.99933	.99610	.98575	.96140	.84179	.61279
	7	1.00000	1.00000	.99993	.99942	.99722	.99051	.94269	.80615
	8	1.00000	1.00000	.99999	.99994	.99961	.99831	.98473	.92700
	9	1.00000	1.00000	1.00000	1.00000	.99996	.99979	.99719	.98071
	10	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99968	.99683
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99976
13	0	.51334	.25419	.12091	.05498	.02376	.00969	.00131	.00012
	1	.86458	.62134	.39828	.23365	.12671	.06367	.01263	.00171
	2	.97549	.86612	.69196	.50165	.33260	.20248	.05790	.01123
	3	.99690	.96584	.88200	.74732	.58425	.42061	.16858	.04614
	4	.99971	.99354	.96584	.90087	.79396	.65431	.35304	.13342
	5	.99998	.99908	.99247	.96996	.91979	.83460	.57440	.29053
	6	1.00000	.99990	.99873	.99300	.97571	.93762	.77116	.50000
	7	1.00000	.99999	.99984	.99875	.99435	.98178	.90233	.70947
	8	1.00000	1.00000	.99998	.99983	.99901	.99597	.96792	.86658
	9	1.00000	1.00000	1.00000	.99998	.99987	.99935	.99221	.95386
	10	1.00000	1.00000	1.00000	1.00000	.99999	.99993	.99868	.98877
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99986	.99829
	12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99988

Table 8 (*continued*)

n	x \ p	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
14	0	.48767	.22877	.10277	.04398	.01782	.00678	.00078	.00006
	1	.84701	.58463	.35667	.19791	.10097	.04748	.00810	.00092
	2	.96995	.84164	.64791	.44805	.28113	.16084	.03979	.00647
	3	.99583	.95587	.85349	.69819	.52134	.35517	.12431	.02869
	4	.99957	.99077	.95326	.87016	.74153	.58420	.27926	.08978
	5	.99997	.99853	.98847	.95615	.88833	.78052	.48585	.21198
	6	1.00000	.99982	.99779	.98839	.96173	.90672	.69245	.39526
	7	1.00000	.99998	.99967	.99760	.98969	.96853	.84986	.60474
	8	1.00000	1.00000	.99996	.99962	.99785	.99171	.94168	.78802
	9	1.00000	1.00000	1.00000	.99995	.99966	.99833	.98249	.91022
	10	1.00000	1.00000	1.00000	1.00000	.99996	.99975	.99609	.97131
	11	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99939	.99353
	12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99994	.99908
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99994
15	0	.46329	.20589	.08735	.03518	.01336	.00475	.00047	.00003
	1	.82905	.54904	.31859	.16713	.08018	.03527	.00517	.00049
	2	.96380	.81594	.60423	.39802	.23609	.12683	.02711	.00369
	3	.99453	.94444	.82266	.64816	.46129	.29687	.09050	.01758
	4	.99939	.98728	.93829	.83577	.68649	.51549	.21728	.05923
	5	.99995	.99775	.98319	.93895	.85163	.72162	.40322	.15088
	6	1.00000	.99969	.99639	.98194	.94338	.86886	.60981	.30362
	7	1.00000	.99997	.99939	.99576	.98270	.94999	.78690	.50000
	8	1.00000	1.00000	.99992	.99922	.99581	.98476	.90495	.69638
	9	1.00000	1.00000	.99999	.99989	.99921	.99635	.96617	.84912
	10	1.00000	1.00000	1.00000	.99999	.99988	.99933	.99065	.94077
	11	1.00000	1.00000	1.00000	1.00000	.99999	.99991	.99807	.98242
	12	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99972	.99631
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99951
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99997
16	0	.44013	.18530	.07425	.02815	.01002	.00332	.00028	.00002
	1	.81076	.51473	.28390	.14074	.06348	.02611	.00329	.00026
	2	.95706	.78925	.56138	.35184	.19711	.09936	.01834	.00209
	3	.99300	.93159	.78989	.59813	.40499	.24586	.06515	.01064
	4	.99914	.98300	.92095	.79825	.63019	.44990	.16657	.03841
	5	.99992	.99670	.97646	.91831	.81035	.65978	.32884	.10506
	6	.99999	.99950	.99441	.97334	.92044	.82469	.52717	.22725
	7	1.00000	.99994	.99894	.99300	.97287	.92565	.71606	.40181
	8	1.00000	.99999	.99984	.99852	.99253	.97433	.85773	.59819
	9	1.00000	1.00000	.99998	.99975	.99836	.99287	.94168	.77275
	10	1.00000	1.00000	1.00000	.99997	.99971	.99843	.98086	.89494
	11	1.00000	1.00000	1.00000	1.00000	.99996	.99973	.99510	.96159
	12	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99906	.98936
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99987	.99791
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99974
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998

Table 8 (*continued*)

n	x	p = 0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
17	0	.41812	.16677	.06311	.02252	.00752	.00233	.00017	.00001
	1	.79223	.48179	.25245	.11822	.05011	.01928	.00209	.00014
	2	.94975	.76180	.51976	.30962	.16370	.07739	.01232	.00117
	3	.99120	.91736	.75561	.54888	.35302	.20191	.04642	.00636
	4	.99884	.97786	.90129	.75822	.57389	.38869	.12600	.02452
	5	.99988	.99533	.96813	.89430	.76531	.59682	.26393	.07173
	6	.99999	.99922	.99172	.96234	.89292	.77522	.44784	.16615
	7	1.00000	.99989	.99826	.98907	.95976	.89536	.64051	.31453
	8	1.00000	.99999	.99970	.99742	.98762	.95972	.80106	.50000
	9	1.00000	1.00000	.99996	.99951	.99690	.98731	.90810	.68547
	10	1.00000	1.00000	1.00000	.99992	.99937	.99676	.96519	.83385
	11	1.00000	1.00000	1.00000	.99999	.99990	.99934	.98941	.92827
	12	1.00000	1.00000	1.00000	1.00000	.99999	.99990	.99748	.97548
	13	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99955	.99364
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99994	.99883
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99986
	16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999
18	0	.39721	.15009	.05365	.01801	.00564	.00163	.00010	.00000
	1	.77352	.45028	.22405	.09908	.03946	.01419	.00132	.00007
	2	.94187	.73380	.47966	.27134	.13531	.05995	.00823	.00066
	3	.98913	.90180	.72024	.50103	.30569	.16455	.03278	.00377
	4	.99845	.97181	.87944	.71635	.51867	.33265	.09417	.01544
	5	.99983	.99358	.95810	.86708	.71745	.53438	.20876	.04813
	6	.99998	.99883	.98818	.94873	.86102	.72170	.37428	.11894
	7	1.00000	.99983	.99728	.98372	.94305	.85932	.56344	.24034
	8	1.00000	.99998	.99949	.99575	.98065	.94041	.73684	.40726
	9	1.00000	1.00000	.99992	.99909	.99458	.97903	.86529	.59274
	10	1.00000	1.00000	.99999	.99984	.99876	.99393	.94235	.75966
	11	1.00000	1.00000	1.00000	.99998	.99977	.99857	.97972	.88106
	12	1.00000	1.00000	1.00000	1.00000	.99997	.99973	.99425	.95187
	13	1.00000	1.00000	1.00000	1.00000	1.00000	.99996	.99872	.98456
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99979	.99623
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99934
	16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99993
	17	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
19	0	.37735	.13509	.04560	.01441	.00423	.00114	.00006	.00000
	1	.75471	.42026	.19849	.08287	.03101	.01042	.00083	.00004
	2	.93345	.70544	.44132	.23689	.11134	.04622	.00546	.00036
	3	.98676	.88500	.68415	.45509	.26309	.13317	.02296	.00221
	4	.99799	.96481	.85556	.67329	.46542	.28222	.06961	.00961
	5	.99976	.99141	.94630	.83694	.66776	.47386	.16292	.03178
	6	.99998	.99830	.98367	.93240	.82512	.66550	.30807	.08353
	7	1.00000	.99973	.99592	.97672	.92254	.81803	.48778	.17964
	8	1.00000	.99996	.99916	.99334	.97125	.91608	.66748	.32380
	9	1.00000	1.00000	.99986	.99842	.99110	.96745	.81391	.50000
	10	1.00000	1.00000	.99998	.99969	.99771	.98946	.91153	.67620
	11	1.00000	1.00000	1.00000	.99995	.99952	.99718	.96477	.82036
	12	1.00000	1.00000	1.00000	.99999	.99992	.99938	.98844	.91647
	13	1.00000	1.00000	1.00000	1.00000	.99999	.99989	.99693	.96822
	14	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99936	.99039
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99990	.99779
	16	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99964
	17	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99996
	18	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Table 9. Random numbers

44	53	73	71	07	11	59	36	46	91	29	97	47	35	58	34	71	60	92	01
54	58	57	28	07	96	51	20	15	18	46	57	41	39	91	92	18	53	10	56
60	25	02	54	51	99	39	65	52	92	70	44	00	96	18	21	05	77	83	99
53	79	36	60	06	19	28	57	98	87	81	21	42	53	93	55	23	58	39	40
02	18	66	31	14	28	98	57	72	52	33	22	60	35	46	07	40	92	83	66
47	16	70	29	93	75	03	00	64	10	57	59	20	39	81	62	97	53	96	36
07	21	03	89	35	84	74	98	42	59	57	18	65	39	77	80	13	90	32	88
74	56	95	22	07	52	14	36	49	72	13	60	11	83	91	99	79	26	21	32
14	19	50	15	51	55	04	76	02	99	80	20	52	95	51	28	63	60	66	54
70	85	49	27	84	39	98	53	92	63	86	10	54	74	62	55	06	65	49	15
59	02	46	96	99	29	26	78	87	61	37	16	59	92	04	38	26	89	24	51
74	93	70	32	63	22	93	38	30	68	13	98	85	99	31	16	15	56	20	72
81	56	27	22	18	93	77	93	37	32	68	92	24	17	74	98	41	64	42	66
22	66	96	25	80	89	35	79	00	77	32	10	43	45	36	71	44	87	69	92
16	70	31	78	43	35	95	08	87	00	10	37	84	57	98	95	44	73	58	87
54	91	63	91	74	28	63	64	40	75	80	34	64	01	13	07	31	46	36	61
02	29	96	52	14	42	18	09	76	51	99	38	76	00	13	17	73	04	09	01
05	57	10	20	81	25	14	79	88	51	95	58	86	52	02	11	13	20	57	67
77	23	59	32	67	02	82	85	41	74	74	82	38	32	58	50	66	44	34	96
13	50	73	99	39	69	26	13	80	93	15	69	47	88	38	71	26	97	69	98
79	38	57	06	40	02	25	24	20	56	72	56	07	43	70	83	60	80	69	59
16	59	91	14	86	47	36	53	55	87	76	54	84	95	60	74	10	65	11	41
43	62	29	60	70	30	86	12	51	84	30	49	21	19	86	04	93	00	52	79
52	70	01	55	59	47	81	86	50	22	16	02	35	62	05	08	01	10	20	84
09	83	11	83	95	73	00	37	69	47	87	48	18	48	44	99	02	92	21	41
58	72	39	07	96	55	99	00	02	84	73	99	37	75	10	61	38	20	33	13
14	87	41	62	53	18	07	86	14	31	79	41	35	16	82	56	25	43	12	35
71	51	45	59	34	60	28	82	49	61	85	34	28	21	13	19	56	83	88	18
95	63	56	84	80	54	42	48	23	24	44	14	09	13	36	62	99	40	65	47
29	96	55	98	10	49	05	20	28	41	88	93	45	90	52	71	95	86	22	84
90	98	95	37	15	68	45	09	33	13	20	56	78	25	44	06	61	71	15	65
69	93	07	12	63	10	67	01	78	12	20	64	25	67	65	03	91	17	73	00
35	80	94	08	95	47	52	58	81	86	44	53	79	39	44	74	50	23	01	76
30	31	38	87	29	29	53	14	35	60	20	34	07	41	87	04	82	63	78	53
70	95	91	85	58	79	00	81	06	31	53	94	50	44	73	55	02	85	60	68
48	96	85	13	47	95	99	44	37	73	23	92	27	98	58	61	05	12	75	31
30	32	24	56	40	51	28	66	44	35	85	66	29	27	70	35	56	43	29	28
60	77	87	75	53	95	43	81	21	39	69	25	24	27	14	61	98	51	02	21
55	78	30	07	80	51	73	01	34	85	78	56	53	74	70	02	48	85	47	44
06	60	81	82	86	16	23	11	25	46	94	19	34	16	93	92	99	68	95	10

Table 9 (*continued*)

80	21	46	27	14	65	72	72	12	69	69	51	59	75	13	44	17	99	50	04
21	34	05	62	82	47	75	21	86	80	35	63	95	50	33	42	44	50	92	87
44	47	11	02	81	83	63	69	16	75	96	41	85	15	34	48	99	71	76	94
43	85	95	60	79	73	38	01	43	29	31	45	45	19	71	97	72	85	31	38
84	45	61	21	18	91	83	77	84	90	95	11	68	84	73	01	32	90	63	03
11	01	89	46	15	84	50	36	32	81	19	50	22	09	44	46	20	60	50	51
38	02	34	71	63	77	69	86	53	01	05	49	47	68	65	92	54	43	47	18
60	71	25	07	27	67	46	14	91	23	36	68	61	08	91	75	25	63	85	14
44	06	67	39	52	28	61	81	61	96	64	44	58	54	50	07	02	18	68	26
30	80	40	29	53	08	66	23	57	43	19	86	02	04	48	73	26	81	75	14
37	43	99	13	24	31	94	51	45	63	07	81	38	31	64	34	14	01	61	15
57	17	72	02	30	96	47	51	68	29	07	91	82	83	53	27	58	02	56	39
57	02	66	81	15	75	06	83	43	89	74	50	26	85	44	61	27	81	65	72
79	70	59	11	96	17	25	84	62	67	37	70	38	34	43	24	61	13	67	75
41	70	06	29	64	47	35	65	92	61	01	37	59	05	29	75	78	45	17	85
34	25	37	45	88	61	39	11	75	45	46	11	08	44	57	52	71	02	44	12
26	34	57	45	08	33	18	83	29	67	27	30	75	77	56	64	06	81	54	41
58	70	45	91	43	13	26	18	46	51	72	02	28	01	31	98	26	56	26	35
49	82	56	18	85	23	93	27	93	95	88	16	10	90	62	15	47	75	14	29
90	93	19	37	00	62	24	94	12	72	52	30	38	91	03	50	63	62	49	33
49	02	40	34	31	03	98	82	20	31	21	83	03	30	85	15	40	16	36	34
67	82	98	70	95	94	69	26	10	96	24	83	73	03	11	96	20	09	52	93
62	81	47	80	85	60	69	37	49	64	66	18	91	85	62	53	21	91	55	46
26	23	18	77	67	99	01	01	78	10	26	48	73	24	92	22	35	84	72	03
27	06	15	45	83	83	44	79	16	81	58	09	11	44	31	15	29	87	09	51
05	04	18	47	52	62	55	07	54	02	94	12	73	42	83	17	45	74	46	50
80	65	81	16	64	58	62	96	60	70	37	44	01	92	29	89	00	93	37	36
42	13	34	66	88	71	70	32	75	59	66	64	12	43	22	67	87	67	12	21
59	46	83	81	65	74	58	75	17	17	38	09	99	19	94	63	72	95	75	90
73	38	05	22	13	15	83	62	78	69	80	90	27	16	47	61	99	50	07	05
02	36	57	26	24	18	72	03	25	11	99	93	41	10	63	24	83	80	61	00
37	65	68	03	64	80	31	86	01	19	74	78	56	48	95	86	18	10	69	88
46	22	42	39	77	76	40	69	22	39	59	83	66	54	74	59	70	38	33	54
71	82	60	96	79	54	52	90	81	97	84	16	36	27	04	29	82	36	35	58
29	92	75	08	63	38	99	59	45	89	54	66	11	39	64	57	71	54	28	38
12	77	19	88	95	25	84	63	56	90	49	91	05	39	10	78	29	15	43	03
88	34	94	21	19	89	88	62	32	47	23	29	13	51	72	04	23	17	83	40
99	37	82	68	50	79	64	92	64	86	05	91	00	73	53	21	96	34	30	10
61	35	36	65	82	19	43	64	20	02	01	62	10	78	33	88	64	14	37	49
67	50	86	22	81	05	30	64	41	72	91	03	59	16	68	44	20	80	34	67

Table 9 (*continued*)

85	82	38	27	90	13	03	53	46	41	28	50	39	09	86	97	31	32	26	50
56	03	71	39	26	90	18	67	26	35	43	14	19	55	93	35	39	42	92	61
97	63	38	94	80	61	05	16	49	52	41	81	39	29	69	18	61	59	88	33
02	84	71	05	21	22	68	88	56	64	00	12	17	78	67	83	90	06	42	79
36	66	74	55	91	48	86	42	65	21	76	43	46	27	18	39	93	83	61	84
42	80	27	08	06	88	15	15	69	46	99	85	92	73	27	74	11	61	22	09
38	77	83	97	59	64	84	17	33	37	72	24	98	90	72	47	63	90	81	91
16	79	74	35	16	75	98	35	01	68	77	20	80	27	09	95	40	94	68	39
42	85	03	08	21	94	36	30	89	87	66	11	32	07	67	29	10	70	87	41
59	69	50	75	91	71	53	37	90	18	71	05	20	27	52	35	14	91	19	57
84	77	70	74	16	27	80	67	72	58	96	90	86	63	42	74	70	08	17	22
10	35	93	15	66	63	20	07	75	83	20	27	48	62	93	81	70	42	11	48
03	41	23	84	82	51	53	16	18	39	99	51	97	47	34	68	81	92	06	22
05	68	40	41	80	39	34	32	92	50	61	74	02	17	69	20	46	33	83	02
35	64	99	24	54	52	93	65	65	16	12	80	04	46	79	99	81	11	94	27
83	52	30	42	70	87	34	01	79	08	31	32	20	82	19	41	79	51	71	06
18	33	36	83	71	19	37	01	51	76	73	65	43	48	17	55	07	51	18	25
81	77	98	06	74	94	57	01	73	44	68	90	39	43	79	29	08	85	14	46
90	36	71	47	71	38	23	12	13	09	20	94	52	41	84	23	97	00	14	05
36	30	83	19	29	48	41	19	13	45	09	36	97	88	87	73	69	21	48	12
88	60	41	05	91	97	89	84	89	00	90	53	68	11	18	91	88	46	21	53
87	98	23	69	73	35	22	43	34	98	92	56	31	05	81	61	99	48	12	89
50	94	84	44	68	85	71	06	16	38	20	30	29	46	56	45	18	75	76	56
69	69	54	43	43	44	39	60	76	92	54	38	79	81	97	80	37	52	43	66
13	24	38	51	40	87	07	66	32	10	49	14	73	34	36	75	24	77	17	03
22	30	15	30	77	62	29	59	75	14	33	69	78	03	75	16	20	22	78	30
14	37	40	15	45	92	36	50	74	03	13	90	36	62	95	66	44	37	80	81
82	69	43	16	12	76	30	25	69	51	67	38	65	59	50	59	86	44	54	69
92	23	27	21	19	88	93	46	79	05	50	48	57	18	71	07	14	43	04	78
87	73	74	90	85	75	79	48	96	90	92	31	79	37	61	95	71	07	10	69
83	67	38	58	01	61	17	42	86	04	97	74	73	89	01	85	74	84	27	80
74	30	48	38	35	44	13	13	92	20	46	37	56	23	45	12	14	97	83	20
26	58	09	14	28	99	46	24	31	87	93	56	21	63	23	88	59	46	84	75
81	27	01	48	98	73	71	10	96	27	67	42	35	06	39	98	52	03	10	07
57	83	78	75	37	90	17	82	53	41	74	81	39	27	74	03	09	17	15	52
46	51	70	08	13	48	89	25	44	00	93	34	52	74	82	39	22	11	29	19
15	30	83	30	66	21	66	01	88	54	78	37	65	69	92	17	59	84	56	96
06	91	94	04	16	58	05	46	75	26	60	01	45	12	10	23	62	09	77	44
36	85	60	66	53	81	33	70	74	14	43	11	36	76	15	19	48	36	46	97
98	34	10	26	45	88	56	60	26	91	07	30	53	10	61	39	09	86	92	67

Table 9 (*continued*)

58	37	49	70	34	55	54	77	49	07	06	92	89	36	79	95	13	58	21	41
60	67	55	59	38	28	81	55	35	85	70	08	11	54	73	73	02	27	11	78
19	73	85	30	48	32	66	07	50	22	04	65	61	36	22	33	92	40	19	16
29	77	67	93	32	64	16	19	38	93	88	24	58	32	82	12	77	21	73	64
56	80	07	33	32	99	08	50	15	46	76	20	92	64	83	21	22	69	77	09
43	53	83	13	65	85	99	38	74	05	97	64	31	31	28	45	72	56	13	11
07	46	52	68	23	44	17	93	81	67	58	43	17	07	97	46	41	31	34	08
40	82	41	08	74	76	67	01	80	06	36	16	68	40	46	59	24	18	70	09
09	59	57	20	60	54	30	23	86	72	88	21	75	53	04	96	86	15	26	00
57	51	78	65	97	27	59	94	93	86	42	68	05	45	75	41	71	96	82	43
00	06	60	78	78	17	85	27	67	48	03	43	02	89	39	57	95	08	91	73
31	47	32	72	71	24	11	06	51	30	52	07	82	34	52	79	50	76	85	02
17	74	99	30	17	21	19	93	62	83	42	96	37	03	42	19	05	97	67	15
00	59	41	15	33	55	62	24	91	28	03	21	35	94	14	61	00	46	17	73
97	51	11	63	13	52	70	09	07	64	41	68	17	82	49	67	53	71	91	12
01	74	41	84	22	08	48	35	52	65	34	97	01	13	00	73	57	94	17	44
79	25	34	64	03	98	75	62	42	80	38	45	79	13	97	90	78	15	00	53
72	78	70	64	74	69	07	27	70	31	82	22	19	79	46	04	58	08	20	02
99	83	05	21	27	47	72	40	02	17	71	15	62	85	25	75	16	19	32	25
51	61	67	44	28	28	76	87	82	11	84	85	25	79	90	40	43	74	66	34
96	12	61	21	20	86	98	30	27	63	77	66	01	85	70	10	06	69	25	15
75	09	67	11	20	70	94	04	27	95	78	72	58	01	69	70	47	80	91	29
06	00	40	52	21	03	91	19	52	05	93	86	36	00	69	82	84	54	16	11
81	09	09	53	88	83	97	63	42	68	01	71	54	31	22	81	08	14	32	73
67	35	80	01	66	85	89	96	16	32	58	62	03	17	59	78	88	60	43	00
07	50	31	57	71	56	23	54	65	19	91	70	51	58	85	60	65	63	28	54
17	03	18	56	94	20	28	47	56	30	08	81	40	25	79	87	56	74	42	70
90	18	69	11	04	75	09	62	32	36	86	56	87	69	97	95	54	15	16	60
93	93	91	05	42	95	47	60	10	86	81	31	84	11	66	95	25	85	52	09
69	02	62	01	26	28	94	77	82	05	22	16	99	51	93	73	13	58	32	78
33	93	37	35	47	98	82	23	15	90	14	99	74	62	57	90	33	81	11	03
79	88	47	16	74	02	14	85	52	15	37	39	25	92	13	82	79	78	16	95
74	88	94	32	47	16	71	23	09	27	46	73	45	65	90	60	17	47	54	41
60	65	60	44	86	86	08	74	79	52	69	11	02	80	94	10	90	63	04	00
54	68	99	86	80	36	53	48	29	86	13	40	37	09	03	92	15	73	21	09
48	19	41	71	98	66	12	31	01	04	56	20	68	02	72	43	84	01	34	79
10	19	90	83	83	47	64	85	13	54	53	88	86	81	32	74	64	45	48	96
81	39	26	85	50	30	63	44	56	59	34	54	60	45	86	70	97	79	43	21
80	29	04	11	93	36	41	21	98	18	04	05	30	68	15	93	01	51	74	89
97	11	53	73	78	65	01	01	80	05	42	01	15	97	73	78	68	85	69	12

Answers to Exercises

201. $\Omega = \{u_0, u_1, \ldots, u_{10}\}$, where u_k denotes "k hens sit on their roosts"
202. (a) $\{2, 3, \ldots, 7\}$; (b) $\{2, 3, 4, \ldots\}$
203. $\{x|0 < x < 3\}$
204. AB: both A and B occur; AB^*: A occurs but not B; A^*B^*: neither A nor B occurs
205. $E = ABC \cup ABD \cup ACD \cup BCD$; $F = ABCD^* \cup ABC^*D \cup AB^*CD \cup A^*BCD$
206. 0.5
207. 0.5
208. 1/55
209. (a) 11/850; (b) 703/1,700; (c) 1/5,525
210. 4/7
211. 24/49
212. (a) 1/120; (b) 1/15; (c) 1/12
213. 1/4
214. (a) 39/49; (b) 143/41,650
215. (a) 0.7; (b) The wanted probability is 9/21, 8/21 and 4/21 for $k = 1, 2,$ and 3
216. (a) 8/11; (b) 4/5; (c) 32/55
217. 3/5
218. (a) $3p(1-p)^2$; (b) $6p^2(1-p)^2/[1-(1-p)^4]$
219. A: $2/n$; B: $2/n$
221. 0.855
222. (a) 0.432; (b) 0.008; (c) 0.992
223. A and B are independent!
224. $(1-p)/(2-p)$
225. $166/6^5$
226. 2,197/20,825
227. (a) $\left[\binom{52}{13} - \binom{48}{13} - \binom{4}{1}\binom{48}{12}\right] \Big/ \left[\binom{52}{13} - \binom{48}{13}\right] = 0.370$;

(b) $1 - \binom{48}{12} \Big/ \binom{51}{12} = 0.561$

228. 37/43

301. $F_X(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{1}{2} & \text{if } a \le x < b \\ 1 & \text{if } x \ge b \end{cases}$

302. 7/9, 1/3, 2/9

303. (a) 1/12; (b) 7/12; (c) 7/12, 1/4

304. (a) 20, 60; (b) $p_X(20) = 1/3$; $p_X(60) = 2/3$

305. (a) $p_X(k) = 0.6^k 0.4$ for $k = 0, 1, \ldots$ (geometric distribution);
(b) 27/125

306. (a) $p_Y(k) = 0.6^{k-1} 0.4$ for $k = 1, 2, \ldots$ (fft-distribution);
(b) 0.6

307. (a) $p_X(k) = \binom{3}{k} 0.4^k 0.6^{3-k}$ for $k = 0, 1, 2, 3$; (b) 44/125

308. (a) $p_X(k) = \binom{4}{k}\binom{6}{3-k} \Big/ \binom{10}{3}$ for $k = 0, 1, 2, 3$; (b) 1/3

309. $(1 - \ln 2)/2 = 0.153$

310. $c = 1/72$

311. $c = 1/(2\sqrt{2})$; $P(X > 0) = 1 - 1/\sqrt{2} = 0.293$

312. $c = 4\beta^{3/2}/\sqrt{\pi}$

313. $\sqrt{2}$

314. $-\frac{1}{3}\ln 2, 0, \ln 2$

315. $-a \ln \ln 2$

316. (a) 2/5; (b) 1/5

317. (a) $1 - e^{-1.2} = 0.699$; (b) $e^{-1.6} = 0.202$; (c) $e^{-1.2} - e^{-1.6} = 0.099$;
(d) $1 - e^{-1.2} + e^{-1.6} = 0.901$; (e) 0

318. $x_\alpha = 0.83, 1.52, 2.15$ for $\alpha = 0.5, 0.1, 0.01$

319. (a) 1/2; (b) 2

320. $x_\alpha = a \ln \dfrac{1-\alpha}{\alpha}$

322. $p_X(k) = (k-1)/2^{k-1}$ for $k = 4, 5, \ldots$

323. $p_X(k) = 1/(n-1)$ for $k = 1, 2, \ldots, n-1$

401. (a) 0.58; (b) 0.11;
(c) $p_X(j) = 0.28, 0.33, 0.29, 0.08, 0.02$ for $j = 0, 1, 2, 3, 4$; $p_Y(k) = 0.20, 0.28, 0.41, 0.11$ for $k = 1, 2, 3, 4$

402. $c = 2$

403. (a) $F_{X,Y}(x, y) = \left(\frac{1}{2} + \frac{1}{\pi}\arctan x\right)\left(\frac{1}{2} + \frac{1}{\pi}\arctan y\right)$
(b) $f_X(x) = \dfrac{1}{\pi(1+x^2)}$; $f_Y(y) = \dfrac{1}{\pi(1+y^2)}$

404. (a) 0.06; (b) 0.18

405.

	k_1	k_2	k_3
j_1	0.03	0.15	0.12
j_2	0.04	0.20	0.16
j_3	0.03	0.15	0.12

406. 1/40
407. 5/9
408. (a) $f_X(x) = \dfrac{1}{(1+x)^2}$ if $x \geq 0$; $f_Y(y) = \dfrac{1}{(1+y)^2}$ if $y \geq 0$.
(b) $F_{X,Y}(x, y) = 1 - \dfrac{1}{1+x} - \dfrac{1}{1+y} + \dfrac{1}{1+x+y}$ if $x \geq 0, y \geq 0$;
$F_X(x) = 1 - \dfrac{1}{1+x}$; $F_Y(y) = 1 - \dfrac{1}{1+y}$
(c) no
409. (a) $f_X(x) = \dfrac{2 + (c+1)x}{c+3} e^{-x}$ if $x \geq 0$; (b) $c = 1$
410. $p_{X,Y}(j, k) = \begin{cases} 4^{j-1}5^{k-j-1}6^{-k} & \text{if } 1 \leq j < k; k = 2, 3, \ldots \\ 4^{k-1}5^{j-k-1}6^{-j} & \text{if } 1 \leq k < j; j = 2, 3, \ldots \end{cases}$
412. 7/8

501. $p_Y(k) = 1/6$ if $k = 2, 4, \ldots, 12$
502. $p_Y(k) = \begin{cases} 1/10 & \text{if } k = 0, 4, 5, 6 \\ 2/10 & \text{if } k = 1, 2, 3 \end{cases}$
503. Y is uniformly distributed over the interval (0, 1)
504. $f_Y(y) = \dfrac{2}{\pi} \cdot \dfrac{1}{1+y^2}$ for $y > 0$, that is $Y = 1/X$ has the same distribution as X
505. $p_Y(k) = (1 - e^{-1/a})e^{-k/a}$ for $k = 0, 1, 2, \ldots$, which is a geometric distribution
506. (a) $p_V(0) = 1/4$; $p_V(1) = 1/2$; $p_V(\sqrt{2}) = 1/4$;
(b) the same distribution as in (a)
507. (a) 0, 1, 2, 3, 4

(b)

k	0	1	2	3	4
number of terms	1	2	3	2	1

(c)

k	0	1	2	3	4
$p_Z(k)$	1/12	2/9	7/18	2/9	1/12

508. $p_{X+Y}(k) = (k+1)p^2q^k$ for $k = 0, 1, 2, \ldots$
509. (a) $p_X(k) = \binom{5}{k} 2^{-5}$ for $k = 0, 1, \ldots, 5$
(b) $p_{X-Y}(k) = \binom{5}{(k+5)/2} 2^{-5}$ for $k = -5, -3, \ldots, 5$

(c) $p_{|X-Y|}(k) = 2\binom{5}{(k+5)/2}2^{-5}$ for $k = 1, 3, 5$

510. $p_Z(k) = \begin{cases} 1/n & \text{if } k = 0 \\ 2(n-k)/n^2 & \text{if } k = 1, 2, \ldots, n-1 \end{cases}$

511. $f_{X+Y}(z) = 6(e^{-2z} - e^{-3z})$ if $z \geq 0$

512. $f_{X+Y}(z) = \frac{1}{2\pi}(\arctan(z+1) - \arctan(z-1))$

513. $f_{X+Y}(z) = \begin{cases} 1 - e^{-z} & \text{if } 0 < z < 1 \\ e^{-z}(e-1) & \text{if } z \geq 1 \end{cases}$

514. $F_{Z_+}(z) = (z/a)^2$ if $0 \leq z \leq a$
$F_{Z_-}(z) = 2z/a - z^2/a^2$ if $0 \leq z \leq a$

515. $f_Z(z) = \sum_1^n \lambda_k \exp(-z \sum_1^n \lambda_k)$ (This is also an exponential distribution)

516. (a) $(11/36)^8 = 0.0000760$; (b) $(25/36)^8 = 0.0541$

517. $f_Z(z) = 2/(1+z)^3$ if $z > 0$

518. $F_Z(z) = \begin{cases} 0 & \text{if } z < 0 \\ (1 - e^{-z/a})(1 - qe^{-z/a})^{-1} & \text{if } z \geq 0 \end{cases}$
$f_Z(z) = (p/a)e^{-z/a}(1 - qe^{-z/a})^{-2}$ if $z \geq 0$

519. 1/4

520. $F_Z(z) = \begin{cases} 0 & \text{if } z < 0 \\ 19/20 & \text{if } z = 0 \\ 1 - \dfrac{(z-1)^2}{20} & \text{if } 0 < z \leq 1 \\ 1 & \text{if } z > 1 \end{cases}$

601. 13/3
602. $2a/3$
604. 1.5
605. 2
606. $a/(a+1)$
607. 1/2
608. $\ln(1+a)$
609. 9, 1/9
610. 3/2, 3/4
611. (a) 2/3, $1/\sqrt{18}$; (b) $6m\sigma - 3\sigma^2 = 2\sqrt{2}/3 - 1/6$
(c) $1 - (m - \sigma)^2 = 1/2 + 2\sqrt{2}/9$ (note that $m + 2\sigma > 1$)
612. 4/45
613. 6.2, 0.05
614. $p_X(0) = 0.5$; $p_X(1) = 0.2$; $p_X(2) = 0.3$;
$p_Y(1) = 0.6$; $p_Y(2) = 0.4$; $E(X) = 0.8$; $E(Y) = 1.4$
The product XY is 0, 1, 2 with probabilities 0.5, 0.2, 0.3;
$E(XY) = 0.8$; $C(X, Y) = -0.32$
615. $E(X) = E(Y) = 15$; $V(X) = V(Y) = 50$; $E(XY) = 200$; $C(X, Y) = -25$; $\rho(X, Y) = -1/2$
617. $V(X) = 1$; $V(Y) = 2/3$; $\rho(X, Y) = 0$; X and Y are uncorrelated but not independent

618. (a) $2\sqrt{2RT/(M\pi)}$; (b) $3RT/(2N)$
619. (a) $1/k + 1 - (1-p)^k$; (b) 5
620. $E(Y) = \int_{-\infty}^{100} x f_X(x)\,dx + \int_{100}^{\infty} (x - 100) f_X(x)\,dx = E(X) - 100(1 - F_X(100))$
has minimum for $m = 109$

701. (a) 0, 1, 4; (b) b_1 is false, b_2 and b_3 are true
702. 0 and $3\sigma/2$
703. -6 and 180
705. 9
706. 36 and $\sqrt{3}$
707. 25
708. 286 and 8.7
709. $E(Y) \approx -1.0, -0.52, 0.48, 1.5$; $D(Y) \approx 0.043$ in all cases
710. 1.0
712. 1/2 and 1/12
713. (a) $E(X) = a + 1/2$; $V(X) = 1/12$; (b) $V_{\text{appr}}(Y) = \frac{1}{12}(a + 1/2)^{-4}$;
(c) $V(Y) = \frac{1}{a(a+1)} - (\ln(1 + 1/a))^2$; 0.19, 0.84, 0.9967

801. 0.0309, 0.197, 0.0309
802. (a) 3.090; (b) -3.090; (c) 1.960; (d) 1.282
803. 2 and 3
804. 1.000, 0.841, 0.159, 0.00135
805. 13.02
806. 0.97725, 0.97722
807. 0.9522; (a) 0.002; (b) 0.006; (c) 0.008
808. 4.98, 0.10
809. 0.20
810. $N(0, 5)$, $N(2, 5)$
811. (a) $N(250, 25)$, $N(50, 25)$, $N(125, 6.25)$;
(b) 0.0694, 0.0228, 0.0455
812. (a) $N(1{,}360, 1{,}525)$, $N(80, 1{,}525)$; (b) 0.153; (c) 0.0202
813. (a) $N(8{,}640, 30^2 \cdot 12)$; (b) 0.9938; (c) 0.0000
814. $N(1, 2)$
815. (a) $N(0, 0.2^2/n)$; (b) 0.317; (c) 0.0455; (d) $n \geq 4{,}331$
816. (a) 0.0124; (b) 0.00616; (c) 27 pills
817. 0.968, 0.724, 0.841
818. (a) 65, 0.2; (b) 0.933
819. 0.159
820. e^b
821. 0.006
822. 0.9995

901. Bin(15, 1/2)
902. (a) 3/4; (b) Bin(5, 3/4); (c) 135/512

903. (a) Bin(8, 1/2); (b) 35/128
904. 0.0706
905. 0.549, 0.198
906. (a) Bin(12, 0.80); (b) Bin(12, 0.20); (c) 0.653; (d) 0.653
907. 0.771
908. (a) Bin(288, 1/3); (b) 96, 64; (c) yes; (d) 0.713, 0.974
909. (a) Bin(48, 1/16); (b) 3, 45/16;
(c) normal approximation should not be used; use Poisson approximation;
(d) 0.647, 0.185
910. (a) 0.652; (b) 0.450; (c) 0.0167
911. (a) Bin(n_A, 0.95), Bin(n_B, 0.90); (b) 0.0104
912. 0.0279
913. $E(X) = 60$ in both cases. $V(X) = npq = 24$ both for $N = 200$ and $N = 1{,}000$; $V(X) = npq\frac{N-n}{N-1} = 24\frac{N-100}{N-1}$, which for $N = 200$ and $N = 1{,}000$ becomes 12.1 and 21.6 (The first variance refers to drawing with replacement, the second to drawing without replacement.)
914. (a) yes; (b) yes; (c) yes; (d) yes
915. (a) 2/3; (b) 0.590 (use binomial approximation)
916. 0.977 (use Poisson approximation)
917. 0.0183
918. 0.132, 0.679, 0.224, 0.137
919. (a) 0.407; (b) 0.331; (c) 0.147
920. 2
921. (a) 0.000; (b) 0.285
922. 0.0769
923. $p_{Y_1,Y_2,Y_3}(k_1, k_2, k_3) = \frac{n!}{k_1!k_2!k_3!}(1/2)^{k_1}(1/6)^{k_2}(1/3)^{k_3}$ where $k_1 + k_2 + k_3 = n$
924. Bin(n, 1/2)
925. $p_X(k) = \binom{2n-k}{n}\left(\frac{1}{2}\right)^{2n-k}, k = 0, 1, \ldots, n$
926. Po(λp)

1101. The arithmetic mean, median, variance, standard deviation, and coefficient of variation are, respectively, 54.0, 52.5, 119, 11 and 20
1102. The arithmetic mean and standard deviation are, respectively, 14.9 and 4.03
1103. The arithmetic mean and standard deviation are, respectively, 174.6 and 32.7
1104. The arithmetic mean and variance are, respectively, 1.78 and 0.2043
1105. The arithmetic mean and standard deviation are, respectively, 9.506 and 0.200

1201. $X \sim N(g, 15^2/2)$
1202. (a) p_1^* can be 0 and 1; p_2^* can be 0, 1/2 and 1; (b) yes;
(c) $V(p_1^*) = pq$; $V(p_2^*) = pq/2$; the efficiency of p_1^* relative to p_2^* is 1/2
1203. (b) $2/n$
1204. (a) 0.82; (b) 0.85
1205. (a) $L(\theta) = \theta(1-\theta)^{4-1} \cdot \ldots \cdot \theta(1-\theta)^{1-1} = \theta^6(1-\theta)^{18}$;
(b) $L(\theta)$ is largest for $\theta = 1/4$; this is the ML estimate

1206. (a) $L(\theta) = \theta^2 \cdot 2.16^{-\theta-1}$; $L(\theta) = 0.40, 0.41$ and 0.34 for $\theta = 2, 3$ and 4;
(b) $L(\theta)$ is largest when $\theta = 3$; this is the ML estimate
1207. The ML estimate is $-n/\sum_1^n \ln x_i$
1208. The ML estimate is $\sum_1^n x_i^2/2n$
1209. (a) $1/(\theta - 1)$; (b) The LS estimate is 3
1210. The LS estimate is 80.77
1211. 1.73
1212. (a) 0.184, 0.232, 0.158; (b) 0.017
1213. 0.067
1214. (a) 0.40; (b) $\sqrt{\lambda}/10$
1215. (a) 0.64; (b) $\sqrt{pq/25}$; (c) 0.096
1216. (a) 0.64; (b) 0.048; (c) 0.066
1217. 80.9, 0.10
1218. $m^* \approx 7.6$; $\sigma^* \approx 2.4$
1219. (a) $\bar{x}^2$; (b) $\bar{x}^2 - \frac{\sigma^2}{n}$
1220. (b) 0.69
1221. $N^* = 5$
1222. $\frac{n}{n-1}x\left(1 - \frac{x}{n}\right)$
1223. (b) $1/n$

1301. (a) 0.21
(b) The number of intervals that miss the target is Bin(15, 0.10). Table 8 shows that $P(X = 0) < P(X = 1) > P(X = 2) > \ldots$; hence it is most probable that one single interval misses the target
1302. (271, 39,500)
1303. $(334, \infty)$
1304. (43.2, 47.2)
1305. (8.04, 8.16)
1306. (7.98, 8.22)
1307. $n \geq 11$
1308. (11.8, 112)
1309. About 200 observations
1310. (0.16, 0.20)
1311. (9.9, 13.7)
1312. (0.12, 0.32)
1313. (5.5, 16.5) (Hence the medicine seems to increase the blood pressure!)
1314. (45.7, 52.9)
1315. (0.024, 0.056)
1316. (a) $n \approx 40{,}000$; (b) $n \approx 6{,}000$
1317. (a) $N \leq$ about 2,500; (b) $n \geq$ about 1,690
1318. (a) (0.024, 0.048); (b) (0, 0.046); (c) (0, 4,600)
1319. (a) 225; (b) 250
1320. (a) (361, 439); (b) (36.1, 43.9)
1321. (a) $2^{-n}, 2^{-n}, 1 - 2^{-n+1}$; (b) $1 - 2^{-n+1}$
1322. (a) 4.425; (b) $\sigma\sqrt{5/12}$; (c) 0.054; (d) (4.30, 4.55)

1323. (1.42, 2.62)
1324. (0.23, 0.30)
1325. $1 - c^{-n}$

1401. (a) Bin(15, 0.5); 0.0592
1402. 20
1403. 0.0115
1404. (a) $a = -1000 \ln(1 - \alpha)$
(b) no, for $x_1 = 75$ is larger than $a = -1000 \ln 0.95 = 51.3$
(c) yes
1405. $P = P(X \leq 45 \text{ if } \theta = 1{,}000) = e^{-45/1{,}000} = 0.044$ which is smaller than 0.05. Hence the result is significant at level of significance 0.05
1406. Since H_1 contains small values of p, person A ought to reject H_0 if it takes a long time before he wins. We obtain $P = \sum_{k=11}^{\infty} 0.2 \cdot 0.8^{k-1} = 0.8^{10} = 0.107$. Since $P > 0.10$, the hypothesis H_0 cannot be rejected.
1407. Only assertion 2
1408. A suitable test quantity is the arithmetic mean $\bar{x}$. The critical region consists of all $\bar{x}$ such that $u = (\bar{x} - 4.0)/(0.2/\sqrt{10}) > \lambda_{0.05} = 1.64$. In this case $\bar{x} = 4.10$ and $u = 1.58$; hence the result is not significant at level of significance 0.05
1409. $h(m) = 1 - \Phi[1.64 - (m - 4.0)/(0.2/\sqrt{10})]$; $h(3.8) \approx 0$; $h(4.3) = 0.9990$
1410. $n \geq 64$
1411. (a) $u = 0.67 < t_{0.05}(9) = 1.83$; H_0 cannot be rejected
(b) $u = -1.24 > -1.83$; H_0 cannot be rejected
1412. $u = -6.36$; we get $|u| > t_{0.025}(17) = 2.11$; H_0 is rejected
1413. $u = 2.76 > t_{0.025}(7) = 2.36$; H_0 is rejected
1414. $u = 11.8$; we get $|11.8| > \lambda_{0.025} = 1.96$; H_0 is rejected
1415. (a) $d(\bar{x}) = 0.0030$
(b) $u = 13.15$; we get $|13.15| > \lambda_{0.025} = 1.96$; H_0 is rejected
1416. $P = (5/6)^{12} = 0.11$; not significant
1417. $P = \sum_{0}^{8} \binom{120}{k} (1/6)^k (5/6)^{120-k} = 0.0024$ (normal approximation); the hypothesis that the die is symmetrical can be rejected at level of significance 0.01
1418. (a) 0.34; (b) 0.036
1419. $26/252 = 0.103$
1420. $P = \sum_{19}^{\infty} e^{-10.0} 10.0^k/k! = 0.0072$ (use Table 7 at the end of the book); H_0 is rejected at level of significance 0.01
1421. $Q = 11.5 > \chi^2_{0.01}(3) = 11.3$; H_0 is rejected
1422. $Q = 3.77 < \chi^2_{0.05}(2) = 5.99$; H_0 cannot be rejected
1423. (a) 0.89; (b) 7 in each sample
1424. $n = 130$. Reject H_0 if the sum of the 130 observations is greater than $520 + 3.09\sqrt{520} = 590.5$
1425. Not significant
1426. 14

1501. (a) $\alpha' + \beta(x + 1) - (\alpha' + \beta x) = \beta$; $\beta^* = 0.7410/1.860 = 0.398$
(b) If $x = \bar{x}$ we get $\alpha + \beta(x - \bar{x}) = \alpha$; $\alpha^* = \bar{y} = 0.537$; $D(\alpha^*) = \sigma/\sqrt{7}$

(c) If $x = 0$ we get $\alpha + \beta(x - \bar{x}) = \alpha - \beta\bar{x}$; $(\alpha - \beta\bar{x})^* = \alpha^* - \beta^*\bar{x} = 0.059$; $D(\alpha^* - \beta^*\bar{x}) = 0.958\sigma$

(d) 0.025

1502. (a) $(-0.21, 1.41)$; (b) $(-0.32, 1.88)$

1503. The hypothesis cannot be rejected

1504. (1.65, 2.57)

1505. The hypothesis is rejected at level of significance 0.05

1506. $v_0^* = y_0/t_0 - t_0 \sum y_i t_i^2 / \sum t_i^4 = 0.313$; (0.290, 0.336)

Answers to Selected Exercises

P1. $p/(7 - 6p)$
P2. Mean f and variance $2f$
P3. 0.206
P4. In both cases the sum assumes the values 2, 3, ..., 12 with probability 1/36, 2/36, 3/36, 4/36, 5/36, 6/36, 5/36, 4/36, 3/36, 2/36, 1/36, respectively
P5. 3
P6. (a) 10; (b) $5x^4$ and $(10/3)(y - y^4)$
P7. 0.85
P8. 15/34
P9. X has mean $1/(\alpha + 3)$ and Y has mean $1 - 1/(\alpha + 3)$
P10. $p_X(2) = 1/10$, $p_X(3) = 3/10$, $p_X(4) = 6/10$
P11. 1/6
P12. 1/4
P13. $(5/6)^{k-2}(1/6)$, $k \geq 2$
P14. (a) $Z = m - X - 30$; (b) $N(m - 30, 7.5^2)$; (c) 492.3
P15. (a) $N(m - 30, 8.08^2)$; (b) 493.3
P16. $\sqrt{m}$ and 1/2
P17. (a) yes; (b) no
P18. $\sqrt{\pi/\beta}/2$ and $(1 - \pi/4)/\beta$
P19.

X \ Y	0	1	2	3	$p_X(j)$
0	$\frac{1}{8}$	$\frac{1}{8}$	0	0	$\frac{1}{4}$
1	0	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$
2	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
$p_Y(k)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

Note that $X \sim \text{Bin}(2, 1/2)$, $Y \sim \text{Bin}(3, 1/2)$

P20. 0.00037
P21. 0.0178
P22. $f_Z(z) = \begin{cases} 4z, & 0 \le z < 1/2 \\ 4(1-z), & 1/2 \le z < 1 \end{cases}$
P23. 1/4
P24. $-1/63 \left(\text{Note that } \iint\limits_{0<x+y<1} x^a y^b (1-x-y)^c \, dx \, dy = a!b!c!/(a+b+c+2)!.\right)$
P25. $E(Z) \approx 0$ and $V(Z) \approx 1/(4n_1) + 1/(4n_2)$
P26. 5/9
P27. (a) 5/6; (b) 29/30; (c) 4/5
P28. 1/2
P29. $F_X(x) = 4x(\sqrt{3} - x)/3, 0 \le x \le \sqrt{3}/2$
P30. (a) 0.0864; (b) 0.173
P31. 0.27
P32. (a) 5/324; (b) 35/648
P33. 0.083
P34. (a) 1/3; (b) 1/5
P35. 0.785
P36. (a) $p_X(k) = \binom{k-1}{r-1} p^r q^{k-r}, k \ge r$; (b) r/p and rq/p^2
P38. $f_Y(y) = y \exp(-y^2/2), y \ge 0$
P39. $n/2^{n-1}$
P40. 11/2

S1. $n \ge 96$
S2. $a^* = -(1/n)\sum \ln x_i$
S3. (a) (1.15, 2.69); (b) (0.68, 1.93)
S4. 8%
S5. $\bar{x}/3$
S6. (37.6, 67.8)
S7. For $a = 0, 1, \ldots, 7$ the power is, respectively, 0, 0, 1/21, 3/21, 6/21, 10/21, 15/21, 1
S8. The hypothesis should be rejected
S9. The ML estimate is $\sum x_i y_i / \sum x_i^2$; its variance is $1/\sum x_i^2$
S10. The ML and LS estimates are both 0.79
S11. $(1/4 + \sum x_i^2/30)^{1/2} - 1/2 = 1.29$
S12. The hypothesis is rejected at the level of significance 1%
S13. (b) (122, 156)
S14. H_0 cannot be rejected at the level of significance 5%
S15. (24.8, 28.9)
S16. H_0 is rejected
S17. (0, 1.78)
S19. (a) Let y be the number of x's equal to 1 or 2. The ML estimate of θ is $y/(2n)$
S20. The hypothesis $d = 0$ is rejected at the level of significance 5%
S21. $c = 3/n$
S22. 30.38
S23. The estimate θ_1^*
S24. 1.83

S25. The assertion cannot be rejected at the level of significance 5%
S26. $h(m) = 1 - \Phi(2 - 4m) + \Phi(-2 - 4m)$
S27. H_0 is rejected at the level of significance 5%
S28. 0.344 and 0.177
S29. $\theta^* = 2.01$; $V(\theta^*) = 0.05\theta^2$
S30. 5.8
S31. (a) 15; (b) 3
S32. 0.891
S33. 590
S34. (a) $h(m) = 1 - \Phi(7.96 - 3m) + \Phi(4.04 - 3m)$; (b) 19
S35. The difference between A and B is significant at the level of significance 5%. (Exclude pairs (H, H) and (H^*, H^*) before performing a sign test.)
S36. (780, 1,220)
S37. (1.589, 1.605)
S38. $n = 15$, $a = 9.73$
S40. $(2n + 1)/(2n) \cdot \max(x_i)$. (Note that the x's are *not* uniformly distributed.)

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