KTH Mathematics

Homework 1 in SF2701 Financial Mathematics, basic course, spring 2018.

Due Tuesday April 24, 2018. Each student should hand in his or her own solutions. Please hand in a paper copy of your solutions either during class, or in the mail box at the student affairs office at Teknikringen 8 D. If you put it in the mail box, please write Camilla Landén on it. Prices should be given with two correct decimals.

---

1. (a) Compute the price of a European call option written on a non-dividend-paying stock. The current stock price is \$100 and the volatility of the stock price is 30%. The maturity of the option is in nine months and the strike price is \$95. The risk free interest rate with continuous compounding is 2% per annum. You should use a three period binomial model to price the option.

Also compute the replicating portfolio.

(b) The same as exercise 1a except you should price and replicate an American call option.

(c) Again, same as exercise 1a except this time you should price and replicate an American put option.

2. (a) Same as exercise 1b except now the underlying stock will pay a dividend of 5% of the stock value in six months time. Note that you do not have to compute the replicating portfolio for this exercise.

(b) Again, same as exercise 1b except now the underlying stock will pay a dividend of \$5 in six months time. Note that you do not have to compute the replicating portfolio for this exercise.

Note that you are supposed to price an American **call** option which means that if early exercise is optimal, it will be so just **before** dividend payment. Make sure that you use the right stock price! (For American **put** options early exercise will always be optimal just **after** dividend payment, so that is why we have not had to think about it.)

3. A stock is expected to pay a dividend of 6% of its value every six months, with the next payment due in one month (then after seven months, and 13 months, and so on). The stock price is \$100, and the risk-free rate of interest is 4% per annum with continuous compounding. An investor has just taken a long position in a nine-month forward contract on the stock.

(a) What are the forward price and the initial value of the forward contract?

(b) Three months later the price of the stock is \$102 and the risk-free rate of interest is still 4% per annum. What are the forward price of a nine-month forward contract on the stock contracted now, and the value of the long position in the forward contract contracted three months ago?

4. Compute the price of an American call option written on a futures contract. The strike price of the option is \$95, the current futures price is \$100, and the time to maturity of the option is nine months. The risk free interest rate with continuous compounding is 2% and the the volatility of the futures price is 30%. As before, you should use a three period binomial model to price the option.

5. Quantlab exercise. Also see the following link:

https://people.kth.se/~aaurell/Teaching/SF2701_VT18/QLintro.html

(a) Pick three stocks from the list in the window "Init data", click on the button "Recalc". Observe the stock prices (shown in the windows with header "Equity") and the corresponding log-returns (shown in the windows with header "Return Equity"). Guess which stock has the highest volatility, and which one has the lowest?

(b) Calculate the standard deviation of the log-returns (see formula on p. 304 in Hull) by clicking on the "Recalc" button in the window "Volatilty calculator".

(c) In the standard Black-Scholes model it is assumed that stock prices are log-normal, i.e. that log-returns are normally distributed. To see if this assumption seems correct pick a stock in the list in the window "Init data", and click on the button "Recalc". You will then see a normalized histogram of log-returns plotted along with a fitted normal distribution. What can you say about the fit? Try a few different stocks and see if there is any difference.

(d) **This is an extracurricular exercise!** It is intended to show that you can also use QuantLab for writing new programs. Open the "Code" section. At the very bottom of the code you will find a function called "find_sharpe_ratio" which has been put as a comment and which is not complete. Your job is to complete it (follow the instructions in the code) and compile it.

Good luck!