KTH Mathematics

Homework 2 in SF2701 Financial Mathematics, basic course, spring 2018.

Due Tuesday May 15, 2018. Each student should hand in his or her own solutions. Please hand in a paper copy of your solutions either during class, or in the black mail box across the hall from the student expedition at the maths department. If you put it in the mail box, please write Camilla Landén on it. Prices should be given with two correct decimals.

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1. In all the exercises below (so exercises 1, 2, 3, and 4) we start out with the Black-Scholes market, i.e. a market consisting of a risk free asset, B, with P-dynamics given by

$$\begin{cases} dB_t &= rB_t dt, \\ B_0 &= 1, \end{cases}$$

and a stock, S, with P-dynamics given by

$$\begin{cases} dS_t &= \alpha S_t dt + \sigma S_t dW_t, \\ S_0 &= s_0. \end{cases}$$

Here W denotes a P-Wiener process and r, α and σ are assumed to be constants.

(a) Compute the price of a European call option written on a non-dividend-paying stock. The current stock price is \$100 and the volatility of the stock price is 30%. The maturity of the option is in nine months and the strike price is \$95. The risk free interest rate with continuous compounding is 2% per annum.

(b) Same as exercise 1a except now the underlying stock will pay a dividend of \$5 in six months time.

(c) Again, same as exercise 1a except now the underlying stock is really a stock index paying a continuous dividend yield of 3%.

2. (a) Consider a six-month European call option on the spot price of gold, that is an option to buy one ounce of gold in six months. The strike price is \$1210, the current spot price of gold is \$1212 per ounce, the six-month futures price of one ounce of gold is \$1215, the risk-free interest rate is 3% per annum, and the volatility of the spot price is 15%, whereas the volatility of the futures price is 16.5%. Compute the price of the European call option.

(b) State an appropriate put-call parity and use it to compute the price of the put option corresponding to the call option in exercise 2a.

3. Suppose that the following parameters have been estimated for the Black-Scholes model: $\alpha = 0.08$, $\sigma = 0.3$, and $r = 0.03$. Furthermore suppose that at time $t = 0$ you sell a European call option on the stock with strike price $K = 105$ and exercise date $T = 1$ year. Your task is to evaluate the performance of a delta hedge for this short position in the option. More precisely, if you sell the option and hedge the position using delta hedging as described below, will you have lost or gained money at time $t = 1$? The hedge should be rebalanced every four months (i.e. at $t = 1/3$ year and so on). Furthermore, the hedge should be set up at time $t = 0$ in such a way that the value of the total portfolio is zero, and the rebalancing should be performed in a self-financing manner.

Below you find the necessary data to compute the hedge.

time t	0	1/3	2/3	1
Stock price	100.00	114.00	104.00	108.00

Make sure to explain how you obtain your numbers!

4. **(Guaranteed equity profits)** Consider a contract written on the UK FTSE 100 index with price process S. The contract has a guaranteed minimum payout and a maximum payout. More precisely, consider a five year contract which pays out 90% times the ratio of the terminal and the initial values of the FTSE, or 130% if otherwise it would be less, or 180% if otherwise it would be more. The claim X is thus given by

$$X = \min\{\max\{1.3, 0.9\tilde{S}_T\}, 1.8\},$$

where T is 5 years and $\tilde{S}_T = S_T/S_0$.

The FTSE (Financial Times/Stock Exchange) 100 index is an index containing the largest 100 companies (by market capitalisation) listed on the London Stock Exchange. As the FTSE 100 index (often referred to as the "Footsie") is composed of 100 different stocks their separate dividend payments will approximate a continuously paying stream. The following data are given

$$\begin{aligned} \text{FTSE drift} \quad \alpha &= 7\% \\ \text{FTSE volatility} \quad \sigma &= 15\% \\ \text{FTSE dividend yield} \quad \delta &= 4\% \\ \text{UK interest rate} \quad r &= 6.5\% \end{aligned}$$

(a) Determine today's arbitrage price of the contract X.
Hint: The payoff of the claim is a piecewise linear function.

(b) What would the price have been if you had forgotten that the constituent stocks of FTSE pay dividends? What would the relative error have been?

5. **Dividends on TeliaSonera**. Quantlab exercise. Also see the following link:

https://people.kth.se/∼aaurell/Teaching/SF2701_VT18/dividends.html

Before you start solving this problem, download and open the workspace and read the workspace guide. Both are available from the course homepage.

Suppose that today is 2016-01-05 and consider the market prices of the forward contracts on TeliaSonera.

(a) Suppose that a dividend of d SEK is to be paid at 2016-04-15. Determine d using the available market data (spot, interest rates, forward prices).

(b) Suppose that the dividend to be paid at time t, corresponding to 2016- 04-15, is αS_t , where $\alpha > 0$ and S_t is the spot price of the TeliaSonera share on 2016-04-15. Determine α using the available market data.

(c) Consider a stock that pays a dividend in the near future, for instance, TeliaSonera which pays a dividend on 2016-04-15. Using the method (a) for determining the dividend, determine the markets expectation of the dividend over a longer time period (a few months). That is, if you consider the TeliaSonera dividend, determine the dividend amount on, say 2015-11-04, 2015-12-04, 2016-01-04, 2016-02-04, 2016-03-04, to investigate how the expected dividend has changed over time. Interpret the result. If you investigate further the markets expected dividend around the date 2016-01-29 you will see something interesting. What?

Good luck!