



KTH Matematik

SOLUTION TO EXAMINATION IN SF2701 FINANCIAL MATHEMATICS 2011-08-23.

Problem 1

The t_2 bond delivers 1 at time t_2 for the price Z_{t_2} today. The same cash flow at time t_2 can also be generated by entering a forward contract on the t_2 -bond for delivery at t_1 and a t_1 -bond with face value $G_0^{(t_1)}$. The payment at time 0 is $Z_{t_1} G_0^{(t_1)}$ in this case. By the law of one price the payments at time 0 must be identical. Therefore

$$G_0^{(t_1)} = \frac{Z_{t_2}}{Z_{t_1}}.$$

Problem 2

The price is 55.2350.

Table 1: Forward tree

time	0	1	2
	6.4600	6.9160	7.4043
		6.0040	6.4278
			5.5801

Table 2: Spot tree

time	0	1	2
	6.4500	6.9107	7.4043
		5.9993	6.4278
			5.5801

Table 3: American option tree

time	0	1	2
	55.2350	91.0676	140.4255
		21.0719	42.7807
			0

Problem 3

The payoff function of the put spread can be written as $(1925 - X)_+ - (1875 - X)_+$ from which we see that the put spread is equivalent to a portfolio with a long put with strike 1925 and a short put with strike 1875. Therefore the price of the put spread is $p(1925) - p(1875)$, where $p(K)$ is the price of a put with strike K . To compute $p(K)$ we use Black's formula. We get

$$\begin{aligned} p(1925) &= 155.0874, \\ p(1875) &= 129.2050, \\ p(1925) - p(1875) &= 25.8824. \end{aligned}$$

Problem 4

The binomial tree for the short rate is given in Table 4. In the caplet tree we must

Table 4: Ho-Lee interest rate tree (%)

period	1	2	3
	2.0	3.4	6.6
		1.4	4.6
			2.6

discount using the interest rate tree. We see that the price of the caplet is 1.6325.

Table 5: Caplet tree (%)

time	0	1	2
	1.6325	2.5323	3.6200
		0.7987	1.6200
			0

Problem 5

Let us assume Black's model. Then the difference between a call and a put is

$$c - p = Z_t(G_0 - K).$$

From this we can extract the forward price G_0 as

$$G_0 = K + \frac{c - p}{Z_t} = 3250 + e^{0.02/3}(205 - 212) = 3242.95$$

With continuous dividends $Z_t G_0 = e^{-\rho t} S_0$, from which the dividend yield can be determined as

$$\rho = -\frac{1}{t} \ln \left(\frac{G_0 Z_t}{S_0} \right) = -3 \ln \left(\frac{3242.95 \cdot e^{-0.02/3}}{3261} \right) = 0.0366.$$

That is, the dividend yield is 3.66% per year.