KTH Matematik

SOLUTION TO EXAMINATION IN SF2701 FINANCIAL MATHEMATICS 2013-08-19.

Problem 1

From the put-call-parity the forward price equals

$$G = \frac{c-p}{Z_T} + K.$$

From the relation between the spot and forward price

$$G = \frac{S_0(1-d)}{Z_T}.$$

Equating the two relations gives

$$d = 1 - \frac{c - p + Z_T K}{S_0} = 1 - \frac{7.75 - 14 + e^{-0.01\cdot 10/12}\cdot 150}{145.5} = 0.0206.$$

Problem 2

(a) First compute the price in euro. The forward price is $G = 28.81016$ and $c = 1.786961$ EUR. The price in SEK is then 15.52 kr.
(b1) With the given numbers of u and d. First compute the price in euro.

Table 1: Forward tree in EUR

time (months)	0	1	2
	28.81	30.73	32.77
		26.89	28.68
			25.10

Table 2: Spot tree in EUR

time (months)	0	1	2
	30.15	30.73	32.77
		26.89	28.68
			25.10

The price in SEK is therefore $1.15 \cdot 8.684 = 9.99$ SEK.

Table 3: Option tree in EUR

time (months)	0	1	2
	1.15	1.89	3.77
		0	0
			0

(b2) With the correct numbers of $u = 1.033321$ and $d = 0.966679$. First compute the price in euro.

Table 4: Forward tree in EUR

time (months)	0	1	2
	28.81	29.77	30.76
		27.85	28.78
			26.92

Table 5: Spot tree in EUR

time (months)	0	1	2
	30.15	29.77	30.76
		27.85	28.78
			26.92

Table 6: Option tree in EUR

time (months)	0	1	2
	1.15	0.88	1.76
		0	0
			0

The price in SEK is therefore $1.15 \cdot 8.684 = 9.99$ SEK.

Problem 3

The present value in EUR of the EUR cash flow is

$$\begin{aligned} P_{EU} &= xe^{-0.0058} + xe^{-2 \cdot 0.0062} + (16 \cdot 10^6 + x)e^{-3 \cdot 0.0066} \\ &= x(e^{-0.0058} + e^{-2 \cdot 0.0062} + e^{-3 \cdot 0.0066}) + 16 \cdot 10^6 \cdot e^{-3 \cdot 0.0066}. \end{aligned}$$

The present value in USD of the US cash flow is $20 \cdot 10^6$ USD which amounts to $16 \cdot 10^6$ EUR. For the currency swap to have present value zero we must have

$$16 \cdot 10^6 = x(e^{-0.0058} + e^{-2 \cdot 0.0062} + e^{-3 \cdot 0.0066}) + 16 \cdot 10^6 \cdot e^{-3 \cdot 0.0066}$$

which leads to

$$x = 16 \cdot 10^6 \frac{1 - e^{-3 \cdot 0.0066}}{e^{-0.0058} + e^{-2 \cdot 0.0062} + e^{-3 \cdot 0.0066}} = 105\,212 \text{ EUR}.$$

Problem 4

First recall (or compute) that $\widehat{E}[e^{az_1}] = e^{a^2/2}$. It follows that

$$\begin{aligned}
Z_k &= \widehat{E}[\exp(-r_1 - \dots r_k)] \\
&= \widehat{E}[\exp(-\theta_1 - \dots - \theta_k - \sigma(k-1)z_1 - \sigma(k-2)z_2 - \dots - z_{k-1})] \\
&= \exp(-\theta_1 - \dots - \theta_k)\widehat{E}[\exp(\sigma(k-1)z_1)]\widehat{E}[\exp(\sigma(k-2)z_2)] \cdots \widehat{E}[\exp(z_{k-1})] \\
&= \exp(-\theta_1 - \dots - \theta_k) \exp\Big(\frac{\sigma^2}{2}[(k-1)^2 + (k-2)^2 + \dots + 1^2]\Big).
\end{aligned}$$

In particular, for $k = 1$, $\theta_1 = -\log(Z_1)$. If the statement holds for k, then it follows that

$$\begin{aligned}
\log Z_{k+1} &= -\theta_1 - \theta_2 - \dots - \theta_{k+1} + \frac{\sigma^2}{2}[(k^2 + (k-1)^2 + \dots + 1^2]) \\
&= \dots = \log(Z_k) - \theta_{k+1} + \frac{\sigma^2}{2}k^2.
\end{aligned}$$

That is, $\theta_{k+1} = \log(Z_k/Z_{k+1}) + \frac{\sigma^2}{2}k^2$. We conclude that the statement holds by induction.

Problem 5

First note that X_t is the forward price in kg gold for one barrel of oil to be delivered at T.
It is sufficient to show that $X_t = E_t^N[X/N]$, which follows if we show that

$$E^N[I_A X/N] = E^N[I_A X_t], \tag{1}$$

for all events A observed at $t \leq T$. Let A be an event observed at t. Consider the strategy to enter, at time t, a forward contract with maturity T to buy one barrel of oil for X_t kg of gold, if A has occurred. If A does not occur do nothing. The strategy costs nothing today and the payoff of the strategy at T is $I_A(X - X_t N)$. Therefore,

$$0 = P_0^{(T)}[I_A(X - X_t N)] = P_0^{(T)}[N]E^N[I_A(X/N - X_t)],$$

and it follows that (1) holds.