KTH Matematik

EXAMINATION IN SF2701 FINANCIAL MATHEMATICS, 2011-08-23, 14:00–19:00.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Interest rates are compounded continuously.

GOOD LUCK!

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General information:

We use the notation $(a)_+ = \max(a, 0)$.

Black's formula for European call and put options:

$$\begin{aligned}
c &= Z_t(G_0^{(t)}\Phi(d_1) - K\Phi(d_2)),\\
p &= Z_t(K\Phi(-d_2) - G_0^{(t)}\Phi(-d_1)),\\
d_1 &= \frac{\ln(G_0^{(t)}/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t},\\
d_2 &= d_1 - \sigma\sqrt{t}
\end{aligned}$$

A table of the standard Normal distribution is given at the end of the exam.

Problem 1

Let Z_t denote the price of a zero-coupon bond with face value 1 and maturity t years from now. Suppose Z_t is known for each future time t.
Derive the forward price $G_0^{(t_1)}[Z_{t_2}]$ for receiving, at time t_1, a zero-coupon bond with maturity t_2. Here $0 < t_1 < t_2$. For full points you must give a complete derivation of the forward price from the law of one price. It is not sufficient just to state the formula. (10 p)

Problem 2

Determine the price of an American option to buy 100 US dollars at 6.00 SEK per dollar. The time to maturity is one year. The value of the US dollar today is 6.45 SEK. The dollars volatility relative to the Swedish crown is 10% in one year. The rate of interest of the Swedish crown is 3% a year, and the one year forward exchange rate is 6.46 SEK for one US dollar. Use a binomial tree to solve the problem, with time interval six months. (10 p)

Problem 3

Let X represent the price in pounds (£) per tonne of Cocoa in March 2012; seven months from now. The volatility of Cocoa is 26% per year. The forward price of X for delivery in seven months is £1915. Use Black's model to compute the price of a put-spread on X with maturity in seven months. The put-spread is a derivative with payoff at maturity given by

$$\begin{cases} 0 & \text{if } X \geq 1925, \\ 1925 - X & \text{if } 1875 \leq X \leq 1925, \\ 50 & \text{if } X \leq 1875. \end{cases}$$

A table of the standard Normal distribution function is given at the end of the exam. The interest rate is 2% per year. (10 p)

Problem 4

Consider Ho-Lee's binomial model where the interest rate between year $t-1$ and t is given by

$$r_t = \ln\left(\frac{Z(0,t-1)}{Z(0,t)}\right) + \ln(\cosh((t-1)\sigma)) + \sigma(b_2 + \cdots + b_t),$$

where $Z(0,t)$ is the price of a zero coupon bond with maturity t (years), and $b_1, b_2, \ldots$ are independent taking values 1 and -1 with probability 0.5 each, under the futures distribution.
The following zero-coupon rates of interest hold: 1-year: 2%, 2-year: 2.2%, 3-year: 3%. The volatility of the one-year rate is assumed to be 1%.
A caplet is a European call option on the short rate. Compute the price of a caplet which pays $100(r_3 - 0.03)_+$ two years from now. (10 p)

Problem 5

An index of shares pays dividends. Your task is to determine the dividend yield per year, assuming the index pays continuous dividends. The value of the index today is 3261. The price of a European call option with strike 3250 and maturity in four months is 205. The price of a European put with strike 3250 and maturity in four months is 212. The four month interest rate is 2% per year. You are encouraged to make model assumptions as long as they don't contradict the given numerical values. State your assumptions clearly. (10 p)

Figure 1: Standard Normal distribution

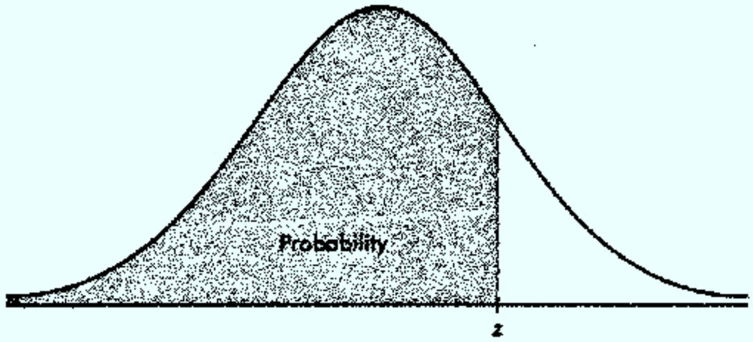

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998