KTH Mathematics

Exam in SF2701 Financial Mathematics.
Wednesday August 15 2018 08.00-13.00.

Examiner: Camilla Landén, tel 070-719 3938.

Aids: Calculator.

General instructions: The solutions should be legible, easy to follow and not lacking in explanations. In particular, all notation used should be explained.

N.B. Number the pages and write your name on each sheet. Write only one exercise per sheet and state the number of the exercise on the sheet.

To pass the exam you need 25 points. A chance to take a supplementary examination is given if you have 23 or 24 points (to take the supplementary examination you must contact Camilla Landén within two weeks from the posting of the results of the exam).

The Black-Scholes formula and a table of the cumulative distribution function of a standard normal can be found at the end of the exam.

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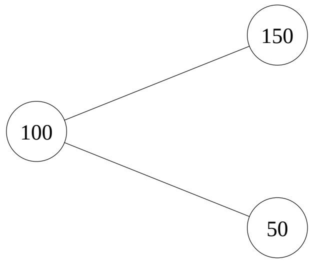
1. (a) Below is a picture of a one-period (time points $t = 0$ and $t = 1$) binomial model with parameters $s_0 = 100$, $u = 1.5$, $d = 0.5$ and $p = 0.75$.



i. What are the arbitrage bounds for the interest rate r? .... (2p)

ii. Given that the price at time $t = 0$ of a European call option with strike price $K = 108$ and exercise time $T = 1$ year has been computed to 22, what is the interest rate r? .... (3p)

(b) Consider the standard Black-Scholes setting. The option strategy to

- hold a long position in (buy) the stock and write (sell) a call option on the same stock

is known as a *covered call*.

i. Suppose that the strike price of the call option is K and the expiry date T. Draw the payoff function of the *covered call*. .... (2p)

ii. Suppose that the stock was initially bought at a price of \$100, whereas the price of the call option initially was \$2 for the strike price \$115. Furthermore suppose that the stock price is still \$100 at the expiry date T. What is the profit from the strategy, i.e how much has the portfolio increased in value? .... (3p)

2. (a) Compute the price of an American put option written on the futures price of crude oil. The current futures price of crude oil is \$50 for one barrel, and the volatility of the futures price is 18%. The maturity of the option is in six months and the strike price is \$52. The risk free interest rate with continuous compounding is 5% per annum. You should use a two period binomial model to price the option. .... (8p)

(b) Estimate the current value of the delta of the option (note that the underlying of the option is the futures price, not the stock itself). .... (2p)

3. (a) A stock is expected to pay a dividend of \$3 of in six months time. The stock price is \$100, and the risk-free rate of interest is 5% per annum with continuous compounding. An investor has just taken a long position in a nine-month futures contract on the stock.

i. What are the futures price and the initial value of the futures contract? .... (3p)

ii. Consider a three-month European put option written on the futures price. The strike price of the option is 105, and the volatility of the futures price is 20%. Compute the price of the futures option. .... (4p)

(b) Consider the standard Black-Scholes setting. Now consider the portfolio corresponding to

- buying a call with strike price K and selling a put with strike price K.

Both options are European, written on the stock, and have the same expiry date T. A portfolio strategy like this is known as a *synthetic long stock*.

Compute delta for a synthetic long stock. .... (3p)

4. (a) Suppose that the current term structure looks as follows:

T	$r(0,T)$
0.5	2.0%
1.0	2.5%
1.5	3.5%
2.0	4.5%
2.5	3.5%
3.0	3.0%

Here the zero rates are quoted per annum with continuous compounding. Compute the prices for the following bonds:

i. A zero coupon bond with face value 100 and maturity six months,

ii. A coupon bond with face value 100, maturity in two years, and a coupon of 4% per annum paid semi-annually,

iii. A coupon bond with face value 100, maturity in three years, and a coupon of 3% per annum paid annually.

. . . . (3p)

(b) Given the term structure in (a), compute the swap rate of a three year fixed-for-floating swap, where six-month LIBOR is to be exchanged for the fixed swap rate R_s. Floating rate payments are to be made semi-annually, whereas the fixed payments are to be made once a year. The notational principal of the swap is one million dollars. The swap rate R_s should be quoted per annum using annual compounding. . . . . (3p)

(c) Company A wishes to borrow pound sterling (GBP) at a fixed rate of interest and Company B wishes to borrow U.S. dollars at a fixed rate of interest. The amounts required by the two companies are roughly the same at the current exchange rate. They have been quoted the following rates per annum (adjusted for differential tax effects):

	USD	GBP
Company A	2.0%	1.3%
Company B	3.5%	2.0%

Design a swap that will net the bank, acting as intermediary, 0.2% per annum. Make the swap equally attractive to both companies and make sure that the bank bears all the foreign exchange risk. . . . . (4p)

Please find Exercise 5 on the next page!

5. For this exercise consider the standard Black-Scholes model.

(a) A *chooser option* is an agreement which gives the owner the right to choose at some prespecified future date T_0, whether the option is to be a call or put option with exercise price K and remaining time to expiry $T - T_0$. Note that K, T and $T_0 < T$ are all prespecified by the agreement. The only thing the owner can choose is whether the option should be a call or a put option, and this choice has to be made at time T_0.
Compute the arbitrage price of the T_0-claim called a chooser option for $t \leq T_0$. .... (5p)
Hint: Start with the price at time T_0. The price can be expressed in terms of the prices of call and put options. Make sure to specify strike prices and expiry dates for the different options!

(b) Determine today's arbitrage price of a contingent T-claim defined by

$$X = \phi(S_T) = \left[\ln\left(\frac{S_T}{S_0}\right)\right]^2.$$

The answer is allowed to be expressed in terms of the parameters of the problem: μ (local mean rate of return of the stock under P), σ, r, T, S_0. .... (5p)

Good luck!

Hints:
You are free to use the following in any of the above exercises.

- The density function of a normally distributed random variable with expectation m and variance σ^2 is given by

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/(2\sigma^2)}.$$

- Let Φ denote the cumulative distribution function for the $\mathrm{N}(0,1)$ distribution. Then

$$\Phi(-x) = 1 - \Phi(x).$$

- The standard Black-Scholes formula for the price $\Pi(t)$ of a European call option with strike price K and time of maturity T is $\Pi(t) = C(t, S(t))$, where

$$C(t,s) = s\Phi[d_1(t,s)] - e^{-r(T-t)}K\Phi[d_2(t,s)].$$

Here Φ is the cumulative distribution function for the $N(0,1)$ distribution and

$$\begin{aligned} d_1(t,s) &= \frac{1}{\sigma\sqrt{T-t}}\left\{\ln\left(\frac{s}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)\right\}, \\ d_2(t,s) &= d_1(t,s) - \sigma\sqrt{T-t}. \end{aligned}$$

The table on the next page shows values of $\Phi(x) = P(X \leq x)$ for $X \in \mathrm{N}(0,1)$, i.e. Φ is the cumulative distribution function of the standard normal distribution. For negative values of x use that

$$\Phi(-x) = 1 - \Phi(x).$$