KTH Matematik

Avd. Matematisk statistik

ELECTIVE HOMEWORK1 in SF2940 PROBABILITY THEORY

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Write your solutions on only one page of each sheet. You should define and explain your notation. Your computations and your line of reasoning should be written down so that they are easy to follow. You will not gain points by submitting an answer without corresponding computations.
Staple your sheets of solutions together, with the homework cover sheet (handed out, downloadable) as uppermost. There can be only one student name on each submitted set of solutions.
THE DEADLINE OF SUBMISSION: THURSDAY THE 17TH OF SEPTEMBER at 12.00 hours.

NO ELECTRONIC SUBMISSION IS PERMITTED.

The homework will be graded and the graded solutions will be handed back NO LATER THAN FRIDAY THE 9TH OF OCTOBER.

There are TEN (10) assignments in Homework1. The maximum number of points awarded by each assignment is conferred next it.

The bonus points gained will be valid **in the exam 28th of October, 2015, AND in the exam 7th of January 2016.**

THE SCALE:
Bonus points in the exam −− graded points in the Homework 1.

0 for 0 – 10 points,
1 for 11 – 20 points ,
2 for 21 – 30 points,
3 for 31 – 40 points,
4 for 41 – 50 points.

Bonus points from Homework2 will be added to the bonus points gained in Homework 1.

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Homework1: Assignments 1.–10.

1. $(\Omega, \mathcal{F}, \mathbf{P})$ is a probability space. $A \in \mathcal{F}$ and $B \in \mathcal{F}$. If the probability that at least one of them occurs is 0.3 and the probability that A occurs but B does not occur is 0.1, what is $\mathbf{P}(B)$? (2 p)

2. Consider the function
$$p(k) = \begin{cases} \frac{k(n-k)}{K}, & k = 1, 2, \ldots, n-1 \\ 0 & \text{elsewhere.} \end{cases}$$
What must the value of K be, in order that $p(k)$ is the probability mass function of some r.v.? (5 p)

3. $(\Omega, \mathcal{F}, \mathbf{P})$ is a probability space, $A \in \mathcal{F}$, and is such that $0 < \mathbf{P}(A) < 1$. Which of the following is an *incorrect* statement?
   a) $\mathbf{P}(\Omega | A) = 1$.
   b) $\mathbf{P}(A | \Omega) = \mathbf{P}(A)$.
   c) $\mathbf{P}(A^c | A) = 0$.
   d) $\mathbf{P}(A | A) = \mathbf{P}(A)$.

   Note that you are required to justify your answer. (1 p)

4. If $\mathbf{P}(A \mid B) \leq \mathbf{P}(A)$, prove that $\mathbf{P}(B \mid A) \leq \mathbf{P}(B)$. (2 p)

5. If $\mathbf{P}(A) = a$ and $\mathbf{P}(B) = b$, prove that $\frac{a+b-1}{b} \leq \mathbf{P}(A \mid B) \leq \frac{a}{b}$. (2 p)

6. $X \in \mathrm{Po}(\lambda)$, $\lambda > 0$. Show that for every $k \in \{0, 1, 2, \ldots\}$ we have
$$\mathbf{P}(X \leq k) = \frac{1}{k!} \int_{\lambda}^{+\infty} e^{-t} t^k dt.$$
(5 p)

7. (X, Y) is a continuous bivariate r.v. with the joint p.d.f.
$$f_{X,Y}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0. \\ 0 & \text{elsewhere.} \end{cases}$$
Let
$$U = X + Y, V = X.$$
   (a) Find the joint p.d.f. of (U, V). (2 p)

   (b) Find the marginal p.d.f.s $f_U(u)$ and $f_V(v)$. Which distributions are these? Are U and V independent? (3 p)

   (c) Find the probability $\mathbf{P}(V \leq 1 \mid U \leq 2)$. (5 p)

8. Solve problem 15 in section 3.8.2. of the Lecture Notes. (10 p)

9. Solve problem 2 (a) and (b) in section 3.8.3 of the Lecture Notes. (10 p)

10. Solve problem 2 (c) in section 3.8.3 in the Lecture Notes. Typing Error in loc.cit of LN: the third subproblem of this exercise should be (c), not (b), as written.

    (3 p)