Errata & Changes: Lecture Notes on on Probability and Random Processes

for

sf2940 Probability Theory Edition 2013

THESE CORRECTIONS/ADDITIONS AND CHANGES HAVE BEEN MADE ON THE THE COURSE HOMEPAGE FILE

- p. 41 Easy Drills
  1. $(\Omega, \mathcal{F}, \mathbf{P})$ is a probability space. $A \in \mathcal{F}$ and $B \in \mathcal{F}$. $\mathbf{P}((A \cup B)^c) = 0.5$ and $\mathbf{P}(A \cap B) = 0.2$. What is the probability that either A or B but not both will occur. **(Answer: 0.3)**
  2. $(\Omega, \mathcal{F}, \mathbf{P})$ is a probability space. $A \in \mathcal{F}$ and $B \in \mathcal{F}$. If the probability that at least one of them occurs is 0.3 and the probability that A occurs but B does not occur is 0.1, what is $\mathbf{P}(B)$? **(Answer: 0.5)** $\rightarrow$ **(Answer: 0.2)** .
- p. 34 " law of the unconscious statisticianindexlaw of the unconscious statistician is extremely useful " should read " law of the unconscious statistician is extremely useful "
- p. 50, Example 2.2.5, eqn. (2.15)

  $$\phi(x) \stackrel{\text{def}}{=} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < +\infty.$$

  should be

  $$\phi(x) \stackrel{\text{def}}{=} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < +\infty.$$

- p. 77 Exercise 6. should be exercise 7. the numbering of the rest of the exercises in section 2.6.2 is not changed.
- p. 80, Exercise 2.6.3.3

  $$f_{X,y}(x,y) = \begin{cases} xe^{-x(1-y)} & x \geq 0, y \geq 0, \\ 0 & \text{elsewhere.} \end{cases}$$

  should be

  $$f_{X,Y}(x,y) = \begin{cases} xe^{-x(1+y)} & x \geq 0, y \geq 0, \\ 0 & \text{elsewhere.} \end{cases}$$

- p. 81, Exercise 2.6.3. 5.1 $\to$ is cancelled
- p. 83, Exercise 2.6.3. 17

  (X, Y) has the p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{(1+x+y)^2} & 0 < x, 0 < y \\ 0 & \text{elsewhere.} \end{cases}$$

  should be (X, Y) has the p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{(1+x+y)^3} & 0 < x, 0 < y \\ 0 & \text{elsewhere.} \end{cases}$$

- p. 83 The exercise 19: **Answers added**

  (X, Y) is a discrete bivariate r.v., such that their joint p.m.f. is

$$p_{X,Y}(j,k) = c\frac{(j+k)a^{j+k}}{j!k!},$$

  where $a > 0$.

  (a) Determine c.

  (b) Find the marginal p.m.f. $p_X(j)$.

  (c) Find $\mathbf{P}(X+Y=r)$.

  (d) Find $E[X]$.

  $\to$

  (a) Determine c. *Answer:* $c = \frac{e^{-2a}}{2a}$

  (b) Find the marginal p.m.f. $p_X(j)$. *Answer:* $p_X(0) = \frac{e^{-a}}{2}$, $p_X(j) = c\frac{a^j}{j!}e^a(j+a)$ for $j \geq 1$.

  (c) Find $\mathbf{P}(X+Y=r)$. *Answer:* $\mathbf{P}(X+Y=r) = c\frac{(2a)^r}{(r-1)!}, r \geq 1, \mathbf{P}(X+Y=0) = 0$.

  (d) Find $E[X]$. *Answer:* $\frac{1}{2}\left(e^{-a}+a+1\right)$.

- p. 96

$$= \int_{A_i} X d\mathbf{P}.$$

  $\to$

$$= \int_{A_j} X d\mathbf{P}.$$

- p. 106 The exercise 3.8.3.1: **Answers added**

(a) Check that

$$\sum_{k=0}^{\infty} \int_{0}^{\infty} f_{X,Y}(k,y)dy = \int_{0}^{\infty} \sum_{k=0}^{\infty} f_{X,Y}(k,y)dy = 1.$$

(b) Compute the mixed moment $E[XY]$ defined as

$$E[XY] = \sum_{k=0}^{\infty} \int_{0}^{\infty} ky f_{X,Y}(k,y)dy.$$

(c) Find the marginal p.m.f. of X.

(d) Compute the marginal density of Y here defined as

$$f_Y(y) = \begin{cases} \sum_{k=0}^{\infty} f_{X,Y}(k,y) & y \in [0,\infty) \\ 0 & \text{elsewhere.} \end{cases}$$

(e) Find

$$p_{X|Y}(k|y) = P(X = k|Y = y), k = 0, 1, 2, \ldots, .$$

(c) Compute $E[X|Y = y]$ and then $E[XY]$ using double expectation. Compare your result with (b).

(a) Check that

$$\sum_{k=0}^{\infty} \int_{0}^{\infty} f_{X,Y}(k,y)dy = \int_{0}^{\infty} \sum_{k=0}^{\infty} f_{X,Y}(k,y)dy = 1.$$

(b) Compute the mixed moment $E[XY]$ defined as

$$E[XY] = \sum_{k=0}^{\infty} \int_{0}^{\infty} ky f_{X,Y}(k,y)dy.$$

Answer: $\frac{2}{\lambda}$.

(c) Find the marginal p.m.f. of X. *Answer*: $X \in \mathrm{Ge}(1/2)$.

(d) Compute the marginal density of Y here defined as

$$f_Y(y) = \begin{cases} \sum_{k=0}^{\infty} f_{X,Y}(k,y) & y \in [0,\infty) \\ 0 & \text{elsewhere.} \end{cases}$$

Answer: $Y \in \mathrm{Exp}(1/\lambda)$.

(e) Find

$$p_{X|Y}(k|y) = P(X = k|Y = y), k = 0, 1, 2, \ldots, .$$

Answer: $X|Y = y \in \mathrm{Po}(\lambda y)$.

(c) Compute $E[X|Y = y]$ and then $E[XY]$ using double expectation. Compare your result with (b).

- p. 145, the kth (descending) factorial moment of X $\to$ the rth (descending) factorial moment of X.

- p. 175 Theorem 6.6.3. If

$$\varphi_{X_n}(t) \to \varphi(t), \quad \text{for all } t,$$

$$\to$$

If $\{\varphi_{X_n}(t)\}_{n=1}^{\infty}$ is a sequence of characteristic functions of random variables X_n, and

$$\varphi_{X_n}(t) \to \varphi(t), \quad \text{for all } t,$$

- p. 185 Show that

$$\sqrt{S_n} - \sqrt{n} \overset{d}{\to} N\left(0, \frac{\sigma^4}{4}\right),$$

should read $\to$

Show that

$$\sqrt{S_n} - \sqrt{n} \overset{d}{\to} N\left(0, \frac{\sigma^2}{4}\right),$$

- p. 212

$$= \frac{\rho^2}{(1-\rho^2)}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2(1-\rho^2)} + \left(\frac{x_2-\mu_2}{\sigma_2^2(1-\rho^2)}\right)^2.$$

should read

$$= \frac{\rho^2}{(1-\rho^2)}\frac{(x_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2(1-\rho^2)} + \frac{(x_2-\mu_2)^2}{\sigma_2^2(1-\rho^2)}.$$

- p.213

Show that for any $\varepsilon 0$ $\to$ Show that for any $\varepsilon > 0$

- Exercise 8.5.1. 14.: Show that

$$\frac{X-Y}{X+Y} \in C(0,1).$$

has **added aid**: *Aid*: Recall the exercise 2.6.3.4..