



KTH Matematik

Avd. Matematisk statistik

SF2940 PROBABILITY THEORY

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# Appendix 2 in A.Gut: An Intermediate Course in Probability. Cover + 2 Pages

## Discrete Distributions

Following is a list of discrete distributions, abbreviations, their probability functions, means, variances, and characteristic functions. An asterisk (\*) indicates that the expression is too complicated to present here; in some cases a closed formula does not even exist.

Distribution, notation	Probability function	$EX$	$\text{Var } X$	$\varphi_X(t)$
One point $\delta(a)$	$p(a) = 1$	$a$	$0$	$e^{ita}$
Symmetric Bernoulli	$p(-1) = p(1) = \frac{1}{2}$	$0$	$1$	$\cos t$
Bernoulli $\text{Be}(p), 0 \leq p \leq 1$	$p(0) = q, p(1) = p; q = 1 - p$	$p$	$pq$	$q + pe^{it}$
Binomial $\text{Bin}(n, p), n = 1, 2, \dots, 0 \leq p \leq 1$	$p(k) = \binom{n}{k} p^k q^{n-k}, k = 0, 1, \dots, n; q = 1 - p$	$np$	$npq$	$(q + pe^{it})^n$
Geometric $\text{Ge}(p), 0 \leq p \leq 1$	$p(k) = pq^k, k = 0, 1, 2, \dots; q = 1 - p$	$\frac{q}{p}$	$\frac{q}{p^2}$	$\frac{p}{1 - qe^{it}}$
First success $\text{Fs}(p), 0 \leq p \leq 1$	$p(k) = pq^{k-1}, k = 1, 2, \dots; q = 1 - p$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^{it}}{1 - qe^{it}}$
Negative binomial $\text{NBin}(n, p), n = 1, 2, 3, \dots, 0 \leq p \leq 1$	$p(k) = \binom{n+k-1}{k} p^n q^k, k = 0, 1, 2, \dots; q = 1 - p$	$n \frac{q}{p}$	$n \frac{q}{p^2}$	$(\frac{p}{1 - qe^{it}})^n$
Poisson $\text{Po}(m), m > 0$	$p(k) = e^{-m} \frac{m^k}{k!}, k = 0, 1, 2, \dots$	$m$	$m$	$e^{m(e^{it} - 1)}$
Hypergeometric $H(N, n, p), n = 0, 1, \dots, N, N = 1, 2, \dots, p = 0, \frac{1}{N}, \frac{2}{N}, \dots, 1$	$p(k) = \frac{\binom{Np}{k} \binom{Nq}{n-k}}{\binom{N}{n}}, k = 0, 1, \dots, Np; q = 1 - p; n - k = 0, \dots, Nq$	$np$	$npq \frac{N-n}{N-1}$	*

## Continuous Distributions

Following is a list of some continuous distributions, abbreviations, their densities, means, variances, and characteristic functions. An asterisk (\*) indicates that the expression is too complicated to present here; in some cases a closed formula does not even exist.

Distribution, notation	Density	$EX$	$\text{Var } X$	$\varphi_X(t)$
Uniform/Rectangular $U(a, b)$	$f(x) = \frac{1}{b-a}, a < x < b$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\frac{e^{itb} - e^{ita}}{it(b-a)}$
$U(0, 1)$	$f(x) = 1, 0 < x < 1$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{e^{it} - 1}{it}$
$U(-1, 1)$	$f(x) = \frac{1}{2},  x  < 1$	$0$	$\frac{1}{3}$	$\frac{\sin t}{t}$
Triangular $\text{Tri}(a, b)$	$f(x) = \frac{2}{b-a} \left( 1 - \frac{2}{b-a} \left  x - \frac{a+b}{2} \right  \right), a < x < b$	$\frac{1}{2}(a+b)$	$\frac{1}{24}(b-a)^2$	$\left( \frac{e^{itb/2} - e^{ita/2}}{\frac{1}{2}it(b-a)} \right)^2$
$\text{Tri}(-1, 1)$	$f(x) = 1 -  x ,  x  < 1$	$0$	$\frac{1}{6}$	$\left( \frac{\sin \frac{t}{2}}{\frac{t}{2}} \right)^2$
Exponential $\text{Exp}(a), a > 0$	$f(x) = \frac{1}{a} e^{-x/a}, x > 0$	$a$	$a^2$	$\frac{1}{1 - ait}$
Gamma $\Gamma(p, a), a > 0, p > 0$	$f(x) = \frac{1}{\Gamma(p)} x^{p-1} \frac{1}{a^p} e^{-x/a}, x > 0$	$pa$	$pa^2$	$\frac{1}{(1 - ait)^p}$
Chi-square $\chi^2(n), n = 1, 2, 3, \dots$	$f(x) = \frac{1}{\Gamma(\frac{n}{2})} x^{\frac{1}{2}n-1} \left( \frac{1}{2} \right)^{n/2} e^{-x/2}, x > 0$	$n$	$2n$	$\frac{1}{(1 - 2it)^{n/2}}$
Laplace $L(a), a > 0$	$f(x) = \frac{1}{2a} e^{- x /a}, -\infty < x < \infty$	$0$	$2a^2$	$\frac{1}{1 + a^2 t^2}$
Beta $\beta(r, s), r, s > 0$	$f(x) = \frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1} (1-x)^{s-1}, 0 < x < 1$	$\frac{r}{r+s}$	$\frac{rs}{(r+s)^2(r+s+1)}$	*

### Continuous Distributions (continued)

Distribution, notation	Density	$EX$	$\text{Var } X$	$\varphi_X(t)$
Weibull $W(\alpha, \beta), \alpha, \beta > 0$	$f(x) = \frac{1}{\alpha\beta} x^{(1/\beta)-1} e^{-x^{1/\beta}/\alpha}, x > 0$	$\alpha^\beta \Gamma(\beta + 1)$	$\alpha^{2\beta} (\Gamma(2\beta + 1) - \Gamma(\beta + 1)^2)$	*
Rayleigh $Ra(\alpha), \alpha > 0$	$f(x) = \frac{2}{\alpha} x e^{-x^2/\alpha}, x > 0$	$\frac{1}{2}\sqrt{\pi\alpha}$	$\alpha(1 - \frac{1}{4}\pi)$	*
Normal $N(\mu, \sigma^2),$ $-\infty < \mu < \infty, \sigma > 0$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2/\sigma^2},$ $-\infty < x < \infty$	$\mu$	$\sigma^2$	$e^{i\mu t - \frac{1}{2}t^2\sigma^2}$
$N(0, 1)$	$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty$	0	1	$e^{-t^2/2}$
Log-normal $LN(\mu, \sigma^2),$ $-\infty < \mu < \infty, \sigma > 0$	$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} e^{-\frac{1}{2}(\log x - \mu)^2/\sigma^2}, x > 0$	$e^{\mu + \frac{1}{2}\sigma^2}$	$e^{2\mu}(e^{2\sigma^2} - e^{\sigma^2})$	*
(Student's) $t$ $t(n), n = 1, 2, \dots$	$f(x) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{\pi n} \Gamma(\frac{n}{2})} \cdot d \frac{1}{(1 + \frac{x^2}{n})^{(n+1)/2}},$ $-\infty < x < \infty$	0	$\frac{n}{n-2}, n > 2$	*
(Fisher's) $F$ $F(m, n), m, n = 1, 2, \dots$	$f(x) = \frac{\Gamma(\frac{m+n}{2}) \Gamma(\frac{m}{2})^{m/2}}{\Gamma(\frac{m}{2}) \Gamma(\frac{n}{2})} \cdot \frac{x^{m/2-1}}{(1 + \frac{mx}{n})^{(m+n)/2}},$ $x > 0$	$\frac{n}{n-2},$ $n > 2$	$\frac{n^2(m+2)}{m(n-2)(n-4)} - \left(\frac{n}{n-2}\right)^2,$ $n > 4$	*

### Continuous Distributions (continued)

Distribution, notation	Density	$EX$	$\text{Var } X$	$\varphi_X(t)$
Cauchy $C(m, a)$	$f(x) = \frac{1}{\pi} \cdot \frac{a}{a^2 + (x-m)^2}, -\infty < x < \infty$	$\beta$	$\beta$	$e^{imt - a t }$
$C(0, 1)$	$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, -\infty < x < \infty$	$\beta$	$\beta$	$e^{- t }$
Pareto $Pa(k, \alpha), k > 0, \alpha > 0$	$f(x) = \frac{\alpha k^\alpha}{x^{\alpha+1}}, x > k$	$\frac{\alpha k}{\alpha - 1}, \alpha > 1$	$\frac{\alpha k^2}{(\alpha - 2)(\alpha - 1)^2}, \alpha > 2,$	*