KTH Matematik

Avd. Matematisk statistik

ELECTIVE HOMEWORK 2 in SF2940 PROBABILITY THEORY

Examiner: Boualem Djehiche, email: boualem@kth.se

Write your solutions on only one page of each sheet. You should define and explain your notation. Your computations and your line of reasoning should be written down so that they are easy to follow. You will not gain points by submitting an answer without corresponding computations. No copier prints will be accepted.

Staple your sheets of solutions together, with the homework cover sheet (downloadable in the course homepage) as uppermost. There can be only one student name on each submitted set of solutions.

The deadline of submission is Thursday October 6, 2016 at 12.00 hours. It should be handed in the Box 'Matematik (SF)' to be found in Teknikringen 8D.

No electronic submission is permitted.

The homework will be graded and the graded solutions will be handed back no later than Friday October 20, 2016.

There are FOUR assignments in Homework 2. The maximum number of points awarded by each assignment is conferred next to it.

The bonus points gained will be valid **in the exam 26th of October, 2016, AND in the Re-exam 20th of December 2016.**

THE SCALE:
Bonus points in the exam −− graded points in the Homework 2.

0 for 0 – 9 points,
5 for 10 – 20 points.

Bonus points from Homework 2 will be added to the bonus points gained in Homework 1. Those who got 10 bonus points in the homeworks may skip Problem 1, whereas those who got 5 points may skip part (a) of Problem 1 in the exam.

Good luck!

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Problem 1

(a) Let $X_n \in N(na, nb^2)$, where $b \neq 0$. Show that

$$\frac{X_n - na}{b\sqrt{n}} \xrightarrow{d} N(0,1), \quad n \to \infty.$$

(2p)

(b) Let $X_1, X_2, \ldots, X_n, \ldots$ be i.i.d. $N(\mu, \sigma^2)$ random variables. Find the asymptotic distribution of $\left(\sqrt{n}(e^{T_n} - e^{\mu})\right)_{n\geq 1}$, where $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. (3p)

Problem 2 Let $X_1, X_2, \ldots, X_n, \ldots$ be independent random variables. $E[X_k] = m^k$, where $m \neq 1,\ k = 1, 2, \ldots$. Let $N \in \text{Po}(\lambda)$ and independent of the variables. Set $S_0 = 0$ and

$$S_N = X_1 + X_2 + \ldots + X_N.$$

Show that

$$E[S_N] = \frac{m}{m-1}\left(e^{\lambda(m-1)} - 1\right).$$

(5p)

Problem 3 $(X_n)_{n\geq 1}$ is a sequence of independent random variables such that

$$P(X_n = \pm 1) = \frac{1}{2}.$$

Let $N \in \text{Po}(\lambda)$ and independent of $(X_n)_{n\geq 1}$. Set $S_N = X_1 + X_2 + \ldots + X_N$. Show that

$$\frac{S_N}{\sqrt{\lambda}} \xrightarrow{d} N(0,1), \quad \lambda \to \infty.$$

(5p)

Problem 4 Assume that the characteristic function of $(X, Y, Z)'$ is

$$\varphi(t, s, u) = \exp\left(2is - s^2 - 2t^2 - 4u^2 - 2st + 2su\right).$$

Compute the conditional distribution of $X + Z$ given $X + Y = 0$. (5p)