EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT

Date: 2014-10-21, 8:00-13:00

Lecturer: Fredrik Armerin, tel. 070-251 75 55, email: armerin@math.kth.se

Allowed technical aids: Calculator.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Good luck!

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Problem 1

A market consists of three risk-free bonds with the following cash flows (CF):

Bond	Price at $t = 0$	CF at $t = 1$	CF at $t = 2$	CF at $t = 3$
1	104	5	5	105
2	98	100	–	–
3	100	2	102	–

(a) Determine the zero rates implied by these bonds. (5p)

(b) Determine the arbitrage free price of the stream of cash flows given by

CF at $t = 1$	CF at $t = 2$	CF at $t = 3$
50	20	75

(2p)

(c) Determine the internal rate of return (IRR) of the stream of cash flows in (b). (3p)

Problem 2

Let S_T be the price of a traded asset at time T. A bank is selling a derivative that gives the owner the amount S_T^β at time T, where $\beta \geq 1$. We assume that

$$\ln(S_T/S_0) \sim N(\mu T, \sigma^2 T)$$

and that the price today of a zero-coupon bond with face value 1 and maturity time T is B_0.

(a) Determine the optimal quadratic hedge of S_T^β consisting of a portfolio of zero-coupon bonds and the traded asset. (4p)

(b) Calculate the variance of the hedging error. (4p)

(c) An individual with utility function

$$u(x) = \ln x$$

has bought the derivative with payoff S_T^β. Determine the certainty equivalent of the payoff in this case. (2p)

Problem 3

A game of football is played between Ipswich and Arsenal. The (decimal) odds given by a bookmaker are

Outcome	Odds
Ipswich	2.5
Arsenal	3.25
Draw	2.85

An investor has utility function

$$u(x) = -\frac{1}{\sqrt{x}}, \quad x > 0,$$

and wants to invest 50 units of currency in the different outcomes of the match.

(a) What is the coefficient of absolute risk aversion for this investor? (1p)

(b) Determine the optimal position for the investor if the investor considers each of the three outcomes of the match equally likely. (6p)

(c) Now assume that the investor can buy and sell the odds contracts and that there is a possibility of saving and lending money at an interest rate of 0 (i.e. $R_0 = 1$). Show that there exists an arbitrage opportunity in this case. (3p)

Problem 4

Consider a market with two risky assets. The mean return vector and covariance matrix of the returns are given by

$$\mu = \begin{bmatrix} 0.10 \\ 0.15 \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} 0.0625 & 0.015 \\ 0.015 & 0.090 \end{bmatrix}$$

respectively. An investor has 100 units of currency to invest.

(a) Determine the minimum variance portfolio if the whole sum is invested. (2p)

(b) Determine the mean and variance of the minimum variance portfolio.(2p)

(c) Determine the portfolio with minimum variance that has mean equal to 0.20 (3p)

(d) Determine the portfolio with minimum variance that has mean 0.20 if there is also the possibility to invest in a bank account with risk-free rate 0.05. (3p)

Problem 5

(a) Define the risk measure Expected shortfall in terms of the Value-at-risk. (2 p)

(b) Calculate the Expected shortfall at level 5% of the random payoff X when X has density function

$$f_X(x) = \frac{1}{2a} e^{-|x|/a}, \; x \in \mathbb{R},$$

for $a > 0$ when $R_0 = 1$. (5 p)

(c) Calculate the Value-at-risk at level 1% of the random payoff X when X fulfills

$$X = \begin{cases} -1\,000 & \text{with probability} & 0.005 \\ -500 & \text{with probabiity} & 0.01 \\ -100 & \text{with probability} & 0.05 \\ 100 & \text{with probability} & 0.9350 \end{cases}$$

and $R_0 = 1$. (3 p)