EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT

*Date:* 2015-01-09, 14:00-19:00

*Lecturer:* Fredrik Armerin, tel. 070-251 75 55, email: armerin@math.kth.se

*Allowed technical aids:* Calculator.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

- - - - - - - - - - - - - - - - - - - -

**Problem 1**

A market consists of the following four risk-free bonds, all with a face value of 100.

- A zero coupon bond with maturity in 1 year and price 98.00 today.
- A coupon bond with maturity in 2 years, a yearly coupon of 3.50 and price 103.80 today.
- A coupon bond with maturity in 3 years, a yearly coupon of 4.00 and price 107.00
- A zero coupon bond with maturity in 5 years and price 94.00 today.

The next coupon payment for all of these bonds is in exactly one year and the coupons are paid out yearly.

(a) Determine the discount factors $d_0, d_1, \ldots, d_5$ implied by these bonds by using linear interpolation when necessary. (6 p)

(b) Determine the value of an annuity paying 50 in years 3, 4 and 5. (2 p)

(c) A stock has price 80 today and a dividend payment in one year of 12. Determine the 2-year forward price $G_0^{(2)}$. (2 p)

**Problem 2**

Let $S_T$ be the price of a traded asset at time $T$. We assume

$$\ln(S_T/S_0) \sim N(\mu T, \sigma^2 T)$$

and that the price today of a zero-coupon bond with face value 1 and maturity time $T$ is $B_0$.

An insurance company has insured $n$ persons. If a person is alive after $T$ years, he receives $S_T$ units of currency. The probability that a person is a alive after $T$ years is $p \in (0, 1)$, and the insured dies independently of each other as well as independently of the price of the traded asset.

(a) Determine the optimal quadratic hedge of the insurance company's liability at $T$ when the zero-coupon bond and the traded asset is used in the hedging portfolio. (3 p)

(b) Now assume that a reinsurance company agrees to construct an asset that pays 1 unit of currency for each of the insured persons who die. Determine the optimal quadratic hedge of the insurance company's liability at $T$ when the zero-coupon bond, the traded asset and the reinsurance contract is used in the hedging portfolio. (7 p)

### Problem 3

An insurance company faces the liability (loss)

$$L = \begin{cases} 0 & \text{with probability} \quad 0.75 \\ Z & \text{with probability} \quad 0.25, \end{cases}$$

where $Z$ is exponentially distributed with mean $\mu > 0$.

(a) Calculate the Value-at-Risk of the liability for any $\alpha \in (0, 1)$ if the interest rate is assumed to be zero. (4 p)

In order to set the insurance premium $\pi$, an insurance company can use the following method. It chooses a utility function $u$ and sets $\pi$ such that

$$E\left[u\left(W + \pi - L\right)\right] = u(W),$$

where $W$ is the initial wealth of the company.

(b) Calculate $\pi$ when the utility function reflects the fact that the insurance company is risk-neutral. (2 p)

(c) Calculate $\pi$ when $u$ is a HARA utility function with $\gamma = 0$ and $\tau > 0$ (i.e. $u$ is an exponential utility function with parameter $\tau$). (4 p)

### Problem 4

Let $X$ be the payoff of a random variable at time 1. For any function $h$ an investor can buy the derivative that pays $h(X)$ at time 1. The forward density of $X$ is denoted $q(x)$ and the investor's own probability density of $X$ is $p(x)$. The forward density is given by

$$q(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \; x \in \mathbb{R},$$

i.e. $X \sim N(\mu, \sigma^2)$ under the forward measure, and the investor's probability function satisfies

$$p(x) = e^{\theta x} q(x), \ x \in \mathbb{R},$$

for a constant $\theta \neq 0$.

(a) Determine the constant $\theta$. (3 p)

(b) Determine the distribution of the random variable $X$ under the investor's probability measure. (3 p)

(c) The investor has utility function

$$u(x) = \ln x$$

and initial capital $V_0$. Determine the optimal derivative position for the investor given the expressions above for $p(x)$ and $q(x)$ when the interest rate is zero. (4 p)

**Problem 5**

(a) A coherent risk measure is a risk measure having four properties. Name these four properties and state them mathematically. (4 p)

(b) Calculate the Expected shortfall at level 2.5% of the random payoff $X$ when $X$ fulfills

$$X = \begin{cases} -1\,000 & \text{with probability} \quad 0.005 \\ -200 & \text{with probabiity} \quad 0.01 \\ 10 & \text{with probability} \quad 0.985 \end{cases}$$

and $R_0 = 1$. (6 p)