EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT

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Allowed technical aids: Calculator.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

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Problem 1

The following (continuously compounded) risk-free zero rates are given:

Maturity (in years)	1	2	3	4	5
Risk-free zero rate	0.010	0.018	0.024	0.028	0.031

On a market the following non-defaultable (i.e. risk-free) coupon bonds are traded:

Bond	Maturity time	Yearly coupon	Face value
1	1	5.00	100
2	2	7.00	200
3	5	5.50	150

The next coupon payment for all these bonds is in exactly one year and the coupons are paid out yearly.

(a) Calculate the price today of the three bonds above. (3 p)

(b) A company has emitted a bond with maturity time in 4 years, face value 5 000 000 and a yearly coupon of 75 000. Since the bond has a risk of defaulting a credit risk premium of 2% has to be added to the risk-free zero rates. Determine the yield-to-maturity (YTM) of this bond.

(3 p)

(c) An insurance company has a liability in 3 years of 10 000 000, and wants to be immune against a parallel shift in the zero rate curve. The investment board of the insurance company has the following demands on the bond portfolio immunizing the liability:

- No more than two bonds should be included in the portfolio.
- Bond number 2 must be included in the portfolio.
- Only long positions in all bonds which are included in the portfolio is allowed.

Calculate the immunization portfolio. (4 p)

Problem 2

(a) Show how it is possible to create an asset with price zero today and cash flow

$$R_{0,T}\,(S_T - F_0)$$

at time T by using a futures contract. Here

$$R_{0,T} = e^{\sum_{k=0}^{T-1} r_{k,k+1}},$$

where the (possibly random) interest rate $r_{k,k+1}$ is the rate over the interval $[k, k+1]$ and this rate is known at time k, F_0 is the futures price at time 0, S_T is the spot price at time T on the asset on which the futures contract is written and the resettlemets times are $0, 1, \ldots, T-1, T$. (4 p)

(b) The price P_T at time T of a barrel of oil has distribution

$$P_T/P_0 \sim U(m - e^{\delta T}, m + e^{\delta T}),$$

i.e. it is uniformly distributed between the values $m - e^{\delta T}$ and $m + e^{\delta T}$, where $m > e^{\delta T}$. A firm has to buy N number of barrels of oil at time T. The amount N is not known today (at time 0), but will be known at time T, and is assumed to satisfy

$$N \sim \text{Po}(\lambda).$$

Hence the total cost of buying the barrels of oil is NP_T.

The price S_T at time T of the stock of an oil company has distribution

$$\ln(S_T/S_0) \sim N(\mu T, \sigma^2 T),$$

and the correlation between S_T and P_T is $\rho \in [0, 1]$. The random variable N is independent of both P_T and S_T. There is also a zero-coupon bond with face value 1, maturity time T and price B_0 today.

Determine the optimal quadratic hedge at time 0 of the liability

$$L = NP_T$$

when a position in the oil company stock and the zero-coupon bond is used in the hedging portfolio. (6 p)

Problem 3

The vector $R = (R_1, R_2, R_3)$ of returns has mean vector

$$\mu = \begin{bmatrix} 1.08 \\ 1.06 \\ 1.12 \end{bmatrix}$$

and covariance matrix

$$\Sigma = \begin{bmatrix} 0.0900 & -0.0225 & 0 \\ -0.0225 & 0.0625 & 0 \\ 0 & 0 & 0.04 \end{bmatrix}.$$

(a) Calculate the minimum variance portfolio for an investor who has initial wealth $V_0 = 1$ and who wants to invest the whole of this amount. (3 p)

(b) Define what is meant by the efficient frontier in the minimization-of-variance approach. (2 p)

(c) Give two examples of efficient portfolios (in the minimization-of-variance sense) in the market $R = (R_1, R_2, R_3)$ given above for an investor with initial wealth $V_0 = 1$ and who is investing the whole amount. (5 p)

Problem 4

An asset with payoff S_T at time $T > 0$ has density under the forward measure given by

$$q(x) = \beta e^{-x\beta}, \ x \geq 0$$

for $\beta > 0$. There is also a zero-coupon bond with maturity time T, face value 1 and price B_0 today.

(a) Determine the price of the call option with strike price K and maturity time T written on the asset, i.e. the payoff at time T is given by

$$X = \max(S_T - K, 0).$$

(4 p)

An investor has utility function

$$u(x) = 2\sqrt{x}, \ x \geq 0,$$

initial wealth V_0 and objective probability density

$$p(x) = \beta^2 x e^{-\beta x}, \ x \geq 0$$

regarding the distribution of S_T at time T. Here the β is the same parameter as in the forward density function $q(x)$.

(b) What is the coefficient of absolute risk aversion for this investor? (1 p)

(c) Determine the optimal derivative position for the investor with utility function given above. (5 p)

Problem 5

(a) Show that the entropic risk measure given by

$$\rho(X) = \frac{\tau}{R_0} \ln E\left[e^{-X/\tau}\right]$$

for some $\tau > 0$ is a convex measure of risk. (5 p)

(b) A project has an initial investment of 1 000 000 and payoff in one year given by

$$\begin{cases} 10\,000\,000 & \text{with probability} \quad 0.20 \\ 2\,000\,000 & \text{with probability} \quad 0.75 \\ 0 & \text{with probability} \quad 0.04 \\ -1\,000\,000 & \text{with probability} \quad 0.01 \end{cases}$$

Calculate the value-at-risk at levels

(i) $p = 0.05$, and

(ii) $p = 0.005$

of this project when $R_0 = 1.05$. (5 p)