EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT

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Allowed technical aids: Calculator.

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

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Problem 1

A market consists of the following risk-free bonds:

Bond	Maturity time (in years)	Coupon rate	Yield-to-maturity
1	1	3.00 %	3.50 %
2	2	4.00 %	4.50 %
3	3	6.00 %	5.00 %

The face value of all bonds is equal to 100, the next coupon payment for all these bonds is in exactly one year and the coupons are paid out yearly. The yield-to-maturity is the internal rate of return of the bond and all rates are denoted using continuous compounding.

(a) Determine the zero rates implied by these bonds. (5p)

(b) Determine the price of a risk-free bond with maturity in 3 years, a coupon rate of 5.50% and a face value of 200. (2p)

(c) A zero coupon bond with face value 100 and maturity in 3 years defaults within the next year with probability 0.025. In the case of a default, the bond pays out 20 % of the face value at time 1, but have no other payoffs. The price today of the defaultable bond is equal to the discounted objective expected values of the this bond's payoffs. Determine this price. (3p)

Problem 2

(a) Consider the problem

$$\min_{A \in \mathcal{A}} E\left[(A-L)^2\right].$$

Here L is a given random variable with finite variance representing a liabilility, and the set $\mathcal{A}$ is the set of all payoffs on the form $h_0 + h^T Z$ where $Z = (Z_1, \ldots, Z_n)$ is a vector of payoffs with invertible covariance matrix Σ_Z and the vector $(h_0, h) \in \mathbb{R}^{n+1}$. Derive the general expression for the minimal expected squared hedging error

$$E\left[(\hat{A} - L)^2\right].$$

You do not have to derive the optimal values of h_0 and h, but your answer should be simplified as much as possible. (5p)

(b) Assume that there exists a market with a zero coupon bond with maturity in T years, face value 1 and price B_0 today and a stock with price S_T at T. The distribution of S_T is given by

$$\ln(S_T/S_0) \sim N(\mu T, \sigma^2 T).$$

Determine the optimal quadratic hedge (h_0, h) and the minimal expected square hedging error in this model when

$$L = S_T^2$$

and both the zero coupon bond as well as the stock is used in the hedging portfolio. (5p)

Problem 3

Consider a model of a market with n assets having gross returns $R = (R_1, R_2, \ldots, R_n)$. The mean vector is given by

$$E[R] = \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{bmatrix}$$

and the covariance matrix by

$$\mathrm{Cov}(R_i, R_j) = \Sigma_{ij} = \begin{cases} 0 & \text{if} \quad i \neq j \\ \sigma_i^2 & \text{if} \quad i = j, \end{cases}$$

where $i, j = 1, \ldots, n$. We assume that every $\sigma_i > 0$. The investor's initial wealth is $V_0 = 1$ and in the problems below all of this must be invested.

(a) Determine the minimum variance portfolio. (3p)

(b) Given any $\gamma > 0$ maximize

$$w^T \mu - \frac{\gamma}{2} w^T \Sigma w.$$

(3p)

(c) Find the portfolio that minimizes the variance, given that the investor wants an expected gross return of μ_0 on his investment. (4p)

Problem 4

In a race at the race track there are five horses competing. The following (decimal) odds are offered by a bookmaker:

Horse	Odds
1	3.75
2	5.75
3	4.5
4	3.5
5	10

An individual has utility function

$$u(x) = \ln(x + 10).$$

(a) Determine the coefficeint of absolute risk aversion. (1p)

(b) Calculate the induvidual's optimal bets if he wants to invest 100 (the whole amount should be used) and his beliefs regarding the winning probabilities are given by

Horse	Winning probability
1	0.25
2	0.15
3	0.10
4	0.45
5	0.05

(6p)

(c) Let V_1 denote the payoff from the race (not including the initial investment of 100) if the optimal strategy from (b) is used. Calculate the certainty equivalent and the absolute risk premium of V_1. (3p)

Problem 5

A firm is considering investing in a project with net payoff X at time 1. The investment costs 100 000, which is paid today (i.e. at time 0), and the investment's payoff V_1 at time 1 is modelled as a random variable with density function

$$f_{V_1}(x) = \frac{1}{a} e^{-x/a}, \ x \geq 0,$$

where

$$a = 150\,000.$$

A risk free zero conpon bond with face value 1 and maturing at time 1 costs $B_0 = 0.96$ today.

(a) Determine the project's value-at-risk at level $p \in (0, 1)$. (5p)

(b) Determine the project's expected shortfall at level $p \in (0, 1)$. (5p)