

Errata - Risk and Portfolio Analysis: Principles and Methods

- Page 7: The equation at the end of the page should read:

$$|\mathbf{k}_2 - \mathbf{k}_1|^2 = |\mathbf{v}_2 + \mathbf{c}_2 - \mathbf{v}_1 - \mathbf{c}_1|^2 = |\mathbf{v}_2 - \mathbf{v}_1|^2 + |\mathbf{c}_2 - \mathbf{c}_1|^2$$

- Page 31: In Exercise 1.5 (b) we assume that the future 1-year zero rate and the 1-year credit spread are independent. It should read: "... and that the 1-year credit spread in 1 year is $N(0.13, 0.03^2)$ -distributed *and independent of the 1-year Treasury zero rate in 1 year*"

- Page 52: The last two sentences should be: Suppose further that the time 1 best estimates of C_k are uncorrelated with the time 1 bond prices and also with pairwise products of bond prices so that

$$\text{Cov}(\mathbb{E}[C_k | \mathbf{I}_1]e^{-r_{1,k}(k-1)}, e^{-r_{1,j+1}j}) = \mathbb{E}[C_k] \text{Cov}(e^{-r_{1,k}(k-1)}, e^{-r_{1,j+1}j})$$

for all j, k .

- Page 54: In the last sentence "Viewed from ... $p_0(t, x)$.", $p_0(t, x)$ should be $p_0(T, x)$
- Page 55: On top of the page $\pi_1(C_T)$ is computed. Here $e^{-r_{1,T-1}(T-1)}$ should be $e^{-r_{1,T}(T-1)}$. This applies also to the two occurrences of $e^{-r_{1,T-1}(T-1)}$ in Example 3.6.
- Page 56: N_1 should be $N_1!$ in the expression

$$\frac{N_1}{c_2! \dots c_k! n_k!} q_2^{c_2} \dots q_k^{c_k} \tilde{q}_k^{n_k}.$$

There are three occurrences of $e^{-r_{1,k-1}(k-1)}$ that should be $e^{-r_{1,k}(k-1)}$.

- Page 58: In the computation of $P(N_2 = k | N_1 = j)$, $e - \lambda$ should be e^λ . In the expression for the value L of the liability,

$$\sum_{k=2}^{\infty} e^{-r_{1,k}(1-\theta)^{k-1}} \quad \text{should be} \quad \sum_{k=2}^{\infty} (e^{-r_{1,k}(1-\theta)})^{k-1}$$

In the expression for the hedging error $\hat{A} - L$ at time 1,

$$\sum_{k=1}^n (\mathbb{E}[C_k] - \mathbb{E}[C_k | \mathbf{I}_1]) e^{-r_{1,k}(k-1)},$$

n should be ∞ .

- Page 70: There should not be a “comma” in the subscript:

$$\sum_{k=1}^m h_k \nabla P_k(\mathbf{r})^T \Delta \mathbf{r} \quad \text{should be} \quad \sum_{k=1}^m h_k \nabla P_k(\mathbf{r})^T \Delta \mathbf{r}$$

- Page 77: There are several typos on this page. The correct version is:

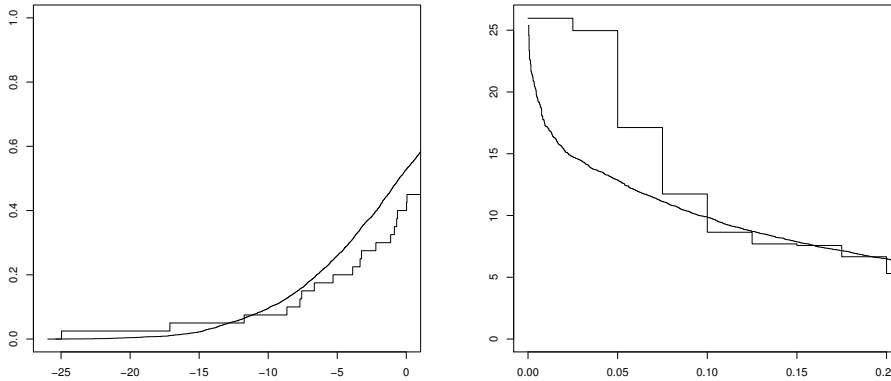
$$\begin{aligned} \nabla P_1(\mathbf{r})^T \mathbf{o}_l &= -105.25 \frac{1}{4} e^{-r_1/4} o_{l1}, \\ \nabla P_2(\mathbf{r})^T \mathbf{o}_l &= -5.5 \frac{3}{4} e^{-r_3 3/4} o_{l3} - 105.5 \frac{7}{4} e^{-r_7 7/4} o_{l7}, \\ \nabla P_3(\mathbf{r})^T \mathbf{o}_l &= -4.5 \frac{2}{4} e^{-r_2 2/4} o_{l2} - \dots - 4.5 \frac{14}{4} e^{-r_{14} 14/4} o_{l14} \\ &\quad - 104.5 \frac{18}{4} e^{-r_{18} 18/4} o_{l18}, \\ \nabla P_4(\mathbf{r})^T \mathbf{o}_l &= -5.0 \frac{4}{4} e^{-r_4 4/4} o_{l4} - \dots - 5.0 \frac{36}{4} e^{-r_{36} 36/4} o_{l36} \\ &\quad - 105.0 \frac{40}{4} e^{-r_{40} 40/4} o_{l40}. \end{aligned}$$

Similarly for the liability:

$$\begin{aligned} P(\mathbf{r}) &= \sum_{k=1}^{40} E[C_k] e^{-r_k k/4}, \\ \nabla P(\mathbf{r})^T \mathbf{o}_l &= - \sum_{k=1}^{40} \frac{k}{4} E[C_k] e^{-r_k k/4} o_{lk}, \quad l = 1, \dots, 4. \end{aligned}$$

- Page 99: In the last paragraph, the maximization-of-expectation problem corresponds to $\sigma_0 = 0.3$ and the minimization-of-variance problem corresponds to $\mu_0 \approx 1.05$.
- Page 124: Exercise 4.6: In “(the probability that the first issuer has rating “Good” and the second issuer has rating “Excellent” is 0.6, etc.)” the value 0.6 should be 0.632718.
- Page 124: Exercise 4.6: The third paragraph begins “Consider an investment ...”. It should be “Consider a 1-year investment ...”.
- Page 125: In the matrix of probabilities, the (3, 1)-entry (row,column) should be 0.064597 instead of 0.064579 and the (4, 1)-entry should be 0.800633 instead of 0.80063.
- Page 129: Just before Remark 5.1: “...omit the second equation in (5.2)” should be “...omit the first equation in (5.2)”
- Page 138: It should be explicitly stated that u in (5.8) is a HARA utility function.

- Page 155: Exercise 5.1 (b) does not make sense. It may read as follows instead:
Another investor is an expected-utility maximizer with utility function $u(x) = x^\beta$ for $\beta < 1$, and invests \$100 in long positions in the defaultable bond and the risk-free bond. Also this investor believes that the default probability is 0.02 and decides to invest less than \$50 dollars in the defaultable bond. What can be said about β ?
- Page 162: In the 18th row, “.at” should be “at”
- Page 164: In the sixth row, the expression $\lambda x_{01} - (1 - \lambda)x_{02}$ should be $\lambda x_{01} + (1 - \lambda)x_{02}$.
- Page 212: The caption of Fig. 7.3 is wrong and should read as follows.
Left plot: the tail of the empirical distribution of X using every 20th observation (rough staircase) and using historical simulation with 5000 resamples (smooth). *Right plot:* estimates of $\text{VaR}_p(X)$ as a function of p based on the empirical distribution using every 20th observation (rough staircase) and using historical simulation with 5000 resamples (smooth).
- Page 212: The plots of Fig. 7.3 are wrong due to an error in the corresponding R-code (now corrected). The correct plots are shown below.



- Page 233: The expression (8.2) should read:

$$\frac{\Gamma((\nu + 1)/2)}{\sigma\sqrt{\nu\pi}\Gamma(\nu/2)} \left(1 + \frac{(x - \mu)^2}{\nu\sigma^2}\right)^{-(\nu+1)/2} \quad \text{for all } x,$$

i.e. there is a σ missing in the denominator.

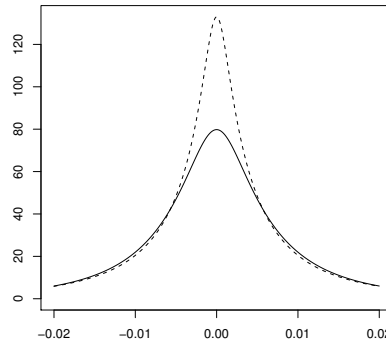
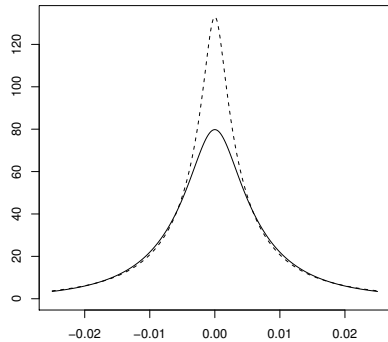
- Page 234: In the last paragraph before Example 8.1 $\text{Exp}(1/\beta)$ should be $\text{Exp}(\beta)$, i.e. “If α is an integer, then $\Gamma(\alpha, \beta)$ is the distribution of the sum of α independent $\text{Exp}(\beta)$ -distributed random variables.”
- Page 235: Just before Example 8.2 it should read: “in the sense that $F_{\mu, \sigma}(x)/F_{\tilde{\mu}, \sigma}(x) \rightarrow \infty$ as $x \rightarrow -\infty$ if $\tilde{\mu} > \mu$.”
- Page 235: Example 8.2: The first math display in Example 8.2 should read:

$$g_\nu\left(\frac{x - \mu}{\sigma}\right) \frac{1}{\sigma} = C \left(1 + \frac{(x - \mu)^2}{\nu\sigma^2}\right)^{-(\nu+1)/2} \quad \text{with} \quad C = \frac{\Gamma((\nu + 1)/2)}{\sigma\sqrt{\nu\pi}\Gamma(\nu/2)}.$$

In the last math display, $\sigma^{\nu+1}$ should be σ^ν , i.e.:

$$t_\nu\left(\frac{x - \mu}{\sigma}\right) \sim \frac{\Gamma((\nu + 1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \nu^{(\nu-1)/2} \sigma^\nu (-x)^{-\nu} \quad \text{as } x \rightarrow -\infty.$$

- Page 241: Example 8.5: “continutation” should be “continuation”
- Pages 246,247: Every θ should be $\boldsymbol{\theta}$ to indicate that the parameter is a vector.
- Page 250: The parameter vector $(\theta_0, \theta_1, \theta_2, \theta_3) = (0, 3, 0, 2.038181)10^{-3}$ does not give a distribution with standard deviation 0.01 and therefore the comparison of the densities in the right plot in Figure 8.6 (the plot to the left below) is not necessarily very informative. With the parameter vector $(\theta_0, \theta_1, \theta_2, \theta_3) = (0, 3, 0, 1.935087)10^{-3}$ we get standard deviation 0.01 and a slightly different plot (the plot to the right below).



- Page 252: The parameter θ_3 should appear as $\theta_3^{1/3}$ in the asymptotic approximation of the left tail of the polynomial normal distribution. The corrected version reads as follows:
Therefore, $G(x) = \Phi(-(-x/\theta_3)^{1/3})$ satisfies $\lim_{p \rightarrow 0} G(F_{\theta}^{-1}(p))/p = 1$. The left tail of G is not explicitly given but it is easier to analyze than that of F_{θ} .

We claim that, as $x \rightarrow -\infty$,

$$\begin{aligned}\Phi(-(-x/\theta_3)^{1/3}) &\sim \phi((-x/\theta_3)^{1/3})(-x/\theta_3)^{-1/3} \\ &= \frac{\theta_3^{1/3}}{\sqrt{2\pi}(-x)^{1/3}} \exp\left\{-\frac{(-x)^{2/3}}{2\theta_3^{2/3}}\right\}.\end{aligned}$$

To show this relation we use l'Hôpital's rule which implies that

$$\lim_{x \rightarrow -\infty} \frac{\Phi(-(-x/\theta_3)^{1/3})}{\phi((-x/\theta_3)^{1/3})(-x/\theta_3)^{-1/3}} = \lim_{x \rightarrow -\infty} \frac{\frac{d}{dx}\Phi(-(-x/\theta_3)^{1/3})}{\frac{d}{dx}\phi((-x/\theta_3)^{1/3})(-x/\theta_3)^{-1/3}}.$$

Computing the derivatives and using that $\phi'(y) = -y\phi(y)$ give

$$\begin{aligned}\frac{d}{dx}\Phi(-(-x/\theta_3)^{1/3}) &= \phi((-x/\theta_3)^{1/3})(-x)^{-2/3}\theta_3^{-1/3}/3, \\ \frac{d}{dx}\phi((-x/\theta_3)^{1/3})(-x/\theta_3)^{-1/3} &= \phi'((-x/\theta_3)^{1/3})(-x)^{-1/3} \\ &\quad + \phi((-x/\theta_3)^{1/3})(-x)^{-4/3}\theta_3^{1/3}/3 \\ &= \phi((-x/\theta_3)^{1/3})(-x)^{-2/3}\theta_3^{-1/3}/3 \\ &\quad + \phi((-x/\theta_3)^{1/3})(-x)^{-4/3}\theta_3^{1/3}/3\end{aligned}$$

which verifies the claim. We conclude that the distribution function F_{θ} of the polynomial normal random variable $\theta_0 + \theta_1 Y + \theta_2 Y^2 + \theta_3 Y^3$ satisfies

$$F_{\theta}(x) \sim \frac{1}{\sqrt{2\pi}(-x/\theta_3)^{1/3}} \exp\left\{-\frac{1}{2}(-x/\theta_3)^{2/3}\right\} \quad \text{as } x \rightarrow -\infty.$$

- Page 256: For any nonnegative independent and identically distributed random variables X, X_1, \dots, X_n it holds that

$$\begin{aligned}\mathrm{P}(X_1 + \dots + X_n > x) &= n \mathrm{P}(X > x) \mathrm{P}(X \leq x)^{n-1} \\ &\quad + \mathrm{P}(X_k > x \text{ and } X_l > x \text{ for some } k \neq l) \\ &\quad + \mathrm{P}(X_1 + \dots + X_n > x \text{ and } X_k \leq x \text{ for every } k),\end{aligned}$$

where the second term on the right-hand side is bounded from above by $\binom{n}{2} \mathrm{P}(X > x)^2$.

- Page 261: The lower bound 1 obviously holds if Y is nonnegative. Here Y is allowed to take negative values so another lower bound is required: Therefore

$$\begin{aligned} \frac{\mathbb{P}(X + Y > x)}{\mathbb{P}(X > x)} &\leq \frac{\mathbb{P}(X > (1 - \varepsilon)x)}{\mathbb{P}(X > x)} + \frac{2\mathbb{P}(|Y| > \varepsilon x)}{\mathbb{P}(X > x)} \\ &\leq \frac{\mathbb{P}(X > (1 - \varepsilon)x)}{\mathbb{P}(X > x)} + \frac{2\mathbb{E}[|Y|^{\alpha+\delta}]}{(\varepsilon x)^{\alpha+\delta} \mathbb{P}(X > x)} \\ &\rightarrow (1 - \varepsilon)^{-\alpha} + 0 \end{aligned}$$

as $x \rightarrow \infty$, where Markov's inequality was used in the second-to-last step above. Since, for any $\lambda > 0$,

$$\mathbb{P}(|Y| > \varepsilon x \mid X > \lambda x) = \frac{\mathbb{P}(X > \lambda x, |Y| > \varepsilon x)}{\mathbb{P}(X > \lambda x)} \leq \frac{\mathbb{P}(|Y| > \varepsilon x)}{\mathbb{P}(X > \lambda x)} \rightarrow 0$$

as $x \rightarrow \infty$ it holds that

$$\begin{aligned} \frac{\mathbb{P}(X + Y > x)}{\mathbb{P}(X > x)} &\geq \frac{\mathbb{P}(X > (1 + \varepsilon)x, |Y| \leq \varepsilon x)}{\mathbb{P}(X > x)} \\ &= \mathbb{P}(|Y| \leq \varepsilon x \mid X > (1 + \varepsilon)x) \frac{\mathbb{P}(X > (1 + \varepsilon)x)}{\mathbb{P}(X > x)} \\ &\rightarrow (1 + \varepsilon)^{-\alpha} \end{aligned}$$

as $x \rightarrow \infty$.

- Page 279: In the proof of Proposition 9.4, the two occurrences of $d\mathbf{z}$ should be removed.
- Page 282: In the last math display,

$$- \lim_{x \rightarrow -\infty} \frac{\Phi(x)}{\phi(x)} \left(\frac{1 - \rho}{1 + \rho} \right)^{1/2}$$

should be

$$- \lim_{x \rightarrow -\infty} \frac{\Phi(x)}{\phi(x)} \phi\left(\left(\frac{1 - \rho}{1 + \rho}\right)^{1/2} x\right) \left(\frac{1 - \rho}{1 + \rho}\right)^{1/2}$$

- Page 289: The last math display is wrong and should read

$$\text{VaR}_p(V_T - V_0/B_0) \approx \text{VaR}_p((\mathbf{w}^T \Sigma \mathbf{w})^{1/2} Y_1) = (\mathbf{w}^T \Sigma \mathbf{w})^{1/2} F_{Y_1}^{-1}(1 - p),$$

- Page 290: The last math display is wrong and should read

$$\begin{aligned} \text{VaR}_p(V_T - V_0/B_0) &\approx T^{1/2} (0.01^2 d (1 + 0.4(d - 1)))^{1/2} \Phi^{-1}(1 - p) \\ &= (0.46T)^{1/2} \Phi^{-1}(1 - p). \end{aligned}$$

- Page 294, second to last row: $N(\mathbf{0}, \Sigma)$ should be $N_n(\mathbf{0}, \Sigma)$
- Page 305: The ML estimates of the Student's t parameters based on British Telecom logreturns should be the parameter estimates based on Deutsche Telekom logreturns and vice versa. It should read

$$\begin{array}{ll}
 (2 \cdot 10^{-4}, 0.015, 7.7) & \text{(British Telecom in pounds),} \\
 (-6 \cdot 10^{-4}, 0.013, 3.7) & \text{(Deutsche Telekom in euros),} \\
 (2 \cdot 10^{-4}, 0.006, 9.6) & \text{(SEK/GBP),} \\
 (8 \cdot 10^{-5}, 0.004, 8.6) & \text{(SEK/EUR).}
 \end{array}$$

- Page 313, first row: the integrand

$$E[I\{Y_1 \leq x_1\} - I\{X_1 \leq x_1\}] E[I\{Y_2 \leq x_2\} - I\{X_2 \leq x_2\}]$$

should be

$$E[(I\{Y_1 \leq x_1\} - I\{X_1 \leq x_1\})(I\{Y_2 \leq x_2\} - I\{X_2 \leq x_2\})]$$

- Page 314, proof of Proposition 9.9: $F_\lambda(x_1, 1)$ and $F_\lambda(1, x_2)$ should be $F_\lambda(x_1, \infty)$ and $F_\lambda(\infty, x_2)$, respectively.
- Page 325, Exercise 9.1 (a): The following should be added at the end of the statement:
“given the constraint that $E[h_0 + h_1 X_1 + \dots + h_d X_d] = E[L]$ ”
- Page 328, Project 10: “The bond portfolio has the cash flow given in Table 9.1.” should be “One share of the bond portfolio has the cash flow given in Table 9.1.”
- Page 329, Project 11: It should be added that Z_k has zero mean:
“that Z_k has a Student's t distribution with three degrees of freedom, zero mean, and standard deviation 0.01 for each k ”