SF2942 - PORTFOLIO THEORY AND RISK MANAGEMENT
FINAL EXAM, THURSDAY OCT 27 2016

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Allowed technical aids: calculator.

All answers must be carefully motivated. Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. A correct answer with no or insufficient explanation will not receive full credit.

Every problem counts for a total of 10 points.

Good luck!

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Problem 1

Principal Component Analysis can be a useful tool in the context of immunization of cash flows. Suppose you have m hedging instruments to be used for the liability L and that the current zero-rate curve is described by $\mathbf{r}$. Furthermore, suppose that you use historical data to estimate the mean vector μ and covariance matrix Σ for changes in the zero-rate curve. Answer the following questions regarding PCA and immunization:

- a) What are the "principal components" and how do you determine them?
- b) How can you decide which and how many of the principal components to use?
- c) Can you give an example of when a PCA would *not* be particularly useful?
- d) Suppose that you have obtained proposed positions in the relevant hedging instruments, from which you can construct an immunization portfolio. Describe a way to try to evaluate the performance of the proposed portfolio using the estimates $\tilde{\mu}$ and $\tilde{\Sigma}$ of the mean vector and covariance matrix.

Motivate your answers properly.

Problem 2

A company will pay out a a bonus to some of its employees, depending on whether they meet some individual performance goals or not, at the end of the year (time T). Each bonus is of size $c(S_T/S_0)$, $c > 0$, where S_t, is the spot price at time t of a share of the parent company's stock. Assume that the spot price follows Black's model: Current spot price is S_0 and we assume that S_T/S_0 has a lognormal distribution with parameters μ, σ.

The (known) number of employees who are eligible for a bonus is N and the probability that an employee meets his or her individual performance goals is estimated to be p;

employees meet their goals independent of each other and of the parent company's share price.

The company wants to construct a hedge against the liability caused by the bonus system. The available instruments are:

- A zero-coupon bond with face value 1, current price B_0 and maturity at T.
- Shares in the stock of the parent company.

Find the optimal quadratic hedge of the liability at time T and compute the hedging error.

Problem 3

Consider a time period from 0 to $T > 0$ and suppose there is a risk-free asset with return R_0 over that period. Suppose that you are investing the known initial capital V_0 at time 0 in a portfolio P that has random future value V at time T.

a) Define the risk measures *Value-at-Risk* and *Expected shortfall* in terms of V_0, V and R_0.

b) Suppose that you are comparing the portfolio P to an alternative portfolio $\tilde{P}$, with future value $\tilde{V}$ (same initial capital used). You know that the Value-at-Risk associated with the portfolio P and the Expected Shortfall associated with $\tilde{P}$, both at level $p = 0.01$, equal some $C > 0$. If you want to use Expected Shortfall to choose a portfolio - less risk is to be preferred - which of the two portfolios would you prefer? Does this answer depend on the (assumed) properties of the distributions of V and $\tilde{V}$? Be as precise as you can.

Problem 4

Consider a time period of length $T > 0$ and suppose that there is a risk-free asset with return R_0 and n risky assets with random returns $R_1, \dots, R_n$ to invest in. A portfolio can be described by the corresponding monetary portfolio weights w_0 and $\mathbf{w} = (w_1, \dots w_n)^\intercal$.

An investor with initial capital V_0 has previously chosen her portfolio according to the investment problem

$$\begin{aligned} \text{maximize} \quad & E\left[V\right] - \frac{c}{2V_0}\mathrm{Var}\left(V\right), \\ \text{subject to} \quad & w_0 + \mathbf{w}^\intercal \mathbf{1} \leq V_0, \end{aligned}$$

where $V = w_0 R_0 + \mathbf{w}^\intercal \mathbf{R}$ and $c > 0$ is a trade-off parameter reflecting the investor's attitude towards risk.

a) Suppose $R_0 = 1.03$ and the risky assets the investor is thinking of investing in are two defaultable bonds. The face value of both bonds are \$1000000 and this is paid to the holder if the issuer of the bond does not default before time T. The current prices are given by

$$P_k = \frac{1000000}{R_0}(1 - p_k), \quad k = 1, 2,$$

where $p_1 = 1/6$ and $p_2 = 1/5$ can be thought of as the market implied probabilities of default. The issuers of the bonds belong to the same sector and can not be considered to default independently of each other. The investor believes that the default probabilities are overestimated and her subjective probability of default is instead $q = 1/10$ for both bonds. Moreover, her subjective view is that the conditional probability that one bond defaults, given that the other has already done so, is $(1 + q)/2$. The investor is only interested in taking long positions in the two bonds. Find the optimal portfolio under these conditions.

b) Consider again the general setting with n risky assets. As a measure of the performance of *any* portfolio $\mathbf{w}$ we can define the *information ratio* $IR(w_0, \mathbf{w})$ by

$$IR(w_0, \mathbf{w}) = \frac{E\left[w_0 R_0 + \mathbf{w}^\intercal \mathbf{R}\right]}{\sqrt{\mathrm{Var}(w_0 R_0 + \mathbf{w}^\intercal \mathbf{R})}}.$$

Suppose the investor is considering choosing her portfolio according to maximizing $IR(w_0, \mathbf{w})$, amongst affordable portfolios. Find the portfolio she would choose if the decision is based on the information ratio rather than the investment problem.
Addition: Describe the portfolio choice for a given (arbitrary) level of risk.

Problem 5

A company is considering buying a full coverage insurance, for the duration of one year, for some of its property. The company is worried about the loss in value of its property due to accident - for example due to extreme weather - and with insurance the value is fully restored. You can assume that there is no other causes for the value to be reduced during the year. The company can be considered a utility-maximizer with utility function $u(x) = x^\gamma$, $x > 0$.

a) What are possible values for γ if the company is to be considered risk-averse?

b) What is the coefficient of absolute risk aversion for the company?

c) In the case of an accident the value of the company's property is estimated to be reduced by a factor $1/2$ and the probability of an accident is estimated to be some $p \in (0, 1)$. Express in terms of γ and p what the company is willing to pay for a full coverage insurance under these assumptions.

d) Suppose now that the type of accidents causing damage to the company's property are categorized as two types. Given that there is an accident, the first type has probability q_1 and causes an estimated loss of 15% in property value, whereas the second type has probability q_2, with $q_1 + q_2 = 1$, and causes a loss of $100U$ % of the property value, where U has a uniform distribution on $[\frac{1}{2}, 1]$. Under these assumptions, what is the company willing to pay for the insurance?