KTH Matematik

EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT, 2012-02-11.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

Define the concept of *the certainty equivalent* and explain how it can be interpreted. In addition you must provide an example of a random variable V_1 (not a deterministic value) representing a future value and a utility function, for which you compute the certainty equivalent. (10 p)

Problem 2

Explain in detail the construction of a swap contract and give an explanation in writing (no math required) why swaps are useful to a life insurance company. (10 p)

Problem 3

You want to bet on the outcome of a football game between Everton and Manchester City. You go to seven different bookmakers and obtain the odds given in Table 1. Is there an arbitrage opportunity? If yes, construct an arbitrage portfolio. (10 p)

Bookmaker	1	2	3	4	5	6	7
Everton	4.30	4.55	4.35	4.30	4.55	4.60	4.70
Draw	3.50	3.55	3.35	3.70	3.30	3.45	3.55
Manchester City	1.85	1.80	1.95	1.80	1.85	1.85	1.75

Table 1: Odds given by seven bookmakers.

Problem 4

You have sold a contract which requires you to pay 100 euros in six months from now. For selling the contract you have received a payment of 97 euros today. On the bond market there are unfortunately no zero-coupon bonds with maturity in six months. There are, however, zero-coupon bonds with face value 100 and maturity in nine months. The nine-month bond costs 95.10 euros. You believe that the three-month zero rate in six months from now, denoted $r_{0.5,0.75}$, follows a Normal distribution with mean 6% (per year) and standard deviation 1.5%. You can invest in the nine-month bond and in a cash account which does not pay interest. Short positions are not allowed.

(a) Among all portfolios that you can afford, determine the portfolio which minimizes the variance subject to the constraint that the expected net value in six months is non-negative. That is, the expected value of the investment portfolio minus the liability is non-negative.

(b) Determine the expected value, and the standard deviation of the optimal portfolio. (10 p)

Problem 5

Consider an investment in a risky bond, issued by a company whose credit rating may change. The value of the bond at the end of the year will depend on the credit rating of the issuer. More precisely, in one year from now the bond has value

$$V_1 = \begin{cases} 1000 & \text{with probability } 0.95, \\ 900 & \text{with probability } 0.045, \\ 0 & \text{with probability } 0.005. \end{cases}$$

The price of the bond today is $V_0 = 940$ and the return of a (one-year) risk-free asset is $R_0 = 1/0.95$.

(a) Compute the Value-at-Risk at level 1% of the net worth $(V_1 - V_0 R_0)$ of a portfolio containing exactly one risky bond.

(b) Compute the Expected Shortfall at level 1% of the net worth $(V_1 - V_0 R_0)$ of a portfolio containing exactly one risky bond.

(10 p)