WHAT YOU SHOULD KNOW BY HEART ABOUT COMBINATORIAL GAME THEORY

- Definitions:
  - $G = \{G^L | G^R\}$,
  - relations: $\geq$, $>$, $=$, $\|$, $\rhd$,
  - $-G$, $G + H$, but the definition of multiplication of numbers is not necessary to remember,
  - short game, (surreal) number, impartial game,
  - the left and right values and sections (no standard notation),
  - the cooled game G_t,
  - the temperature $t(G)$ and the mean value G_∞.
- Special notation:
  - the game $* = \{0|0\}$,
  - the nimbers $*n$ for positive integers n,
  - the mex of a set of nonnegative integers,
  - the nim addition operator $\oplus$.
- General lingo:
  - dyadic rationals,
  - normal play convention, misère play convention.
- Theorems (but you don't have to remember their proofs in detail):
  - Conway induction
  - Fundamental properties of the relations $\geq$, $>$, $=$, $\|$, $\rhd$ and the minus and plus operators.
  - Numbers are totally ordered (ONAG, Th. 2).
  - The number tree (ONAG, page 11) for short games (finite birthdays).
  - The (general) simplicity theorem (ONAG, Th. 11).
  - The translation theorem (ONAG, Th. 90)
  - Increasing an option weakly increases the game (ONAG, Th. 67(i)).
  - Games can be simplified if they have dominated or reversible options (ONAG, Th. 68).
  - Any short game has a unique canonical form (ONAG, Th. 69).
  - Linearity of cooling (ONAG, Th. 64)
  - Properties of temperature and mean value (ONAG, Th. 66 and Th. 59).
  - Computing the left and right values (ONAG, Th. 56).
  - Grundy's theorem: Every short impartial game is equivalent to a nimber (ONAG, Th. 71; Lecture notes 3, Th. 6.3).
  - Addition of nimbers: $*m + *n = *(m \oplus n)$ (Lecture notes 3, Th. 6.2).
- Abilities
  - Compute the value of any nim position and any Blue-Red Hackenbush tree position.
  - Draw the thermograph of any short game.
  - Write any short game in canonical form.
  - Perform a minimax search with alpha-beta pruning in a game tree.