SF2972 GAME THEORY
Problem Set 1
Due January 27, at the lecture

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1. Find all Nash equilibria, in pure and mixed strategies, in each of the three games in Lecture 1, Section 3.4.

2. Find all Nash equilibria, in pure and mixed strategies, in the partnership game in Lecture 1, Section 6.3

3. Write up the normal form of Games 1-4 in Lecture 2 (Section 1). For each of these normal-form games, find all Nash equilibria, in pure and mixed strategies. (Watch out in Game 4: there may be more NE than you first think!]

4. Consider the following 2×2 normal-form game G, for arbitrary $a, b > 0$:

	H	T
H	$a, 0$	$0, b$
T	$0, b$	$1, 0$

   (a) Find all pure and mixed Nash equilibria in G.

   (b) For each pure or mixed Nash equilibrium in G, and each player, find the player's set of pure and mixed best replies to the equilibrium in question.

5. Reconsider the Cournot oligopoly game in Lecture 2, Section 6.1. Let there be n firms in the market, where n is an arbitrary positive integer (including the case $n = 1$). For each firm, let its strategy set be $[0, 100]$, let $Q = q_1, ..., q_n$ and let the payoff function of each firm be its profit, defined as

$$\pi_i(q_1, ..., q_n) = \begin{cases} (100 - Q) \cdot q_i & \text{if } Q \leq 100 \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i = 1, 2, ...n$$

   (a) Which of the hypotheses in Theorem 7.1 in Lecture 2 are met, and which are not? [Note that the case $n = 1$ is special.]

   (b) Find a Nash equilibrium (in pure strategies), for each $n \in \mathbb{N}$. Is it unique?

   (c) Compute aggregate supply, in (your) Nash equilibrium, $Q^{NE}(n)$, as a function of n. Explain its dependence on n, the number of firms in the market, and explain also how each firm's output depends on n.

6. Two individuals, 1 and 2, contribute to a public good (say, a clean shared kitchen) by making individual efforts $x \in [0, 1]$ and $y \in [0, 1]$. The resulting level of the public good is $x + y$. Individual utilities are given by

$$u_1(x, y) = (x + y)\, e^{-x} \quad \textit{and} \quad u_2(x, y) = (x + y)\, e^{-y}$$

Each individual strives to maximize his or her expected utility.

(a) Game A: Suppose both effort levels are chosen simultaneously. Which of the hypotheses in Theorem 7.1 in Lecture 2 are met, and which are not? Does this game have a pure-strategy Nash equilibrium? Find all its pure-strategy Nash equilibria.

(b) Game B: Suppose individual 1 first chooses her effort level, and that this is observed by individual 2, who then chooses his effort level. Solve it by backward induction. [First find 2's optimal effort level, for any given effort level chosen by 1, and then find 1's optimal effort level when 1 anticipates 2's subsequent choice.]

(c) Write up the normal form of Game B. Prove that your backward-inductive solution constitutes a Nash equilibrium. Show that this game has infinitely many pure-strategy Nash equilibria. In particular, show that there exist a Nash equilibrium in which individual 2 makes effort $x_2 = 1/2$.