GAME THEORY — PROBLEM SET 2
DUE FEB 10 AT THE LECTURE

Problem 1

We are playing a game of Nim (with the normal play convention) and my last move resulted in a position with five piles of sizes 26, 19, 10, 9, 7. Do you have a winning move from this position? If so, what is the maximum number of sticks you can remove in a winning move?

Problem 2

Let $G = \{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$ be an impartial game as represented in the lecture notes.

(a) How many positions does G have?
(b) Draw the directed acyclic graph that represents G.
(c) Compute the Grundy value $g(G)$.
(d) Who will win the game?

Problem 3

Do the exercise on page 77 in ONAG, that is, show that $\{0|1\} + \{0|\frac{1}{2}\} = \{\frac{1}{2}|1\}$ by a strategic discussion.

Problem 4

Find the "one-line proofs" that Conway omits in ONAG Chapter 1, namely, show directly from the definitions that

(a) $-(x+y) \equiv (-x) + (-y)$,
(b) $-(-x) \equiv x$

for any games x and y, and that

(c) $x0 \equiv 0$,
(d) $x1 \equiv x$,
(e) $xy \equiv yx$,
(f) $(-x)y \equiv x(-y) \equiv -xy$

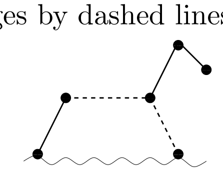
for any numbers x and y.

Problem 5

Show that the game $\{0|*\}$ is positive but less than any positive number!

Problem 6

Compute the value of the following Blue-Red Hackenbush position. Blue edges are shown by solid lines and red edges by dashed lines. The ground is a wavy line.



Problem 7

Compute the value of the following Blue-Red Hackenbush position. Who will win the game?