

## WHAT YOU SHOULD KNOW BY HEART ABOUT COMBINATORIAL GAME THEORY

- Definitions:
  - $G = \{G^L|G^R\}$ ,
  - relations:  $\geq, >, =, \parallel, \triangleright$ ,
  - $-G, G + H$ , but the definition of multiplication of numbers is not necessary to remember,
  - short game, (surreal) number, impartial game,
  - the left and right values and sections (no standard notation),
  - the cooled game  $G_t$ ,
  - the temperature  $t(G)$  and the mean value  $G_\infty$ .
- Special notation:
  - the game  $*$  =  $\{0|0\}$ ,
  - the numbers  $*n$  for positive integers  $n$ ,
  - the mex of a set of nonnegative integers,
  - the nim addition operator  $\oplus$ .
- General lingo:
  - dyadic rationals,
  - normal play convention, misère play convention.
- Theorems (but you don't have to remember their proofs in detail):
  - Conway induction
  - Fundamental properties of the relations  $\geq, >, =, \parallel, \triangleright$  and the minus and plus operators.
  - Numbers are totally ordered (ONAG, Th. 2).
  - The number tree (ONAG, page 11) for short games (finite birthdays).
  - The (general) simplicity theorem (ONAG, Th. 11).
  - The translation theorem (ONAG, Th. 90)
  - Increasing an option weakly increases the game (ONAG, Th. 67(i)).
  - Games can be simplified if they have dominated or reversible options (ONAG, Th. 68).
  - Any short game has a unique canonical form (ONAG, Th. 69).
  - Linearity of cooling (ONAG, Th. 64)
  - Properties of temperature and mean value (ONAG, Th. 66 and Th. 59).
  - Computing the left and right values (ONAG, Th. 56).
  - Grundy's theorem: Every short impartial game is equivalent to a number (ONAG, Th. 71; Lecture notes 3, Th. 6.3).
  - Addition of numbers:  $*m + *n = *(m \oplus n)$  (Lecture notes 3, Th. 6.2).
- Abilities
  - Compute the value of any nim position and any Blue-Red Hackenbush tree position.
  - Draw the thermograph of any short game.
  - Write any short game in canonical form.