GAME THEORY — PROBLEM SET 3

Problem 1

We are playing a game of Nim (with the normal play convention) and my last move resulted in a position with five piles of sizes 26, 19, 10, 9, 7. Do you have a winning move from this position? If so, what is the maximum number of sticks you can remove in a winning move?

Problem 2

Let $G = \{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$ be an impartial game as represented in the lecture notes.

(a) How many positions does G have?
(b) Draw the directed acyclic graph that represents G.
(c) Compute the Grundy value $g(G)$.
(d) Who will win the game?

Problem 3

Find the "one-line proofs" that Conway omits in ONAG Chapter 1, namely, show directly from the definitions that

(a) $-(x+y) \equiv (-x) + (-y)$,
(b) $-(-x) \equiv x$

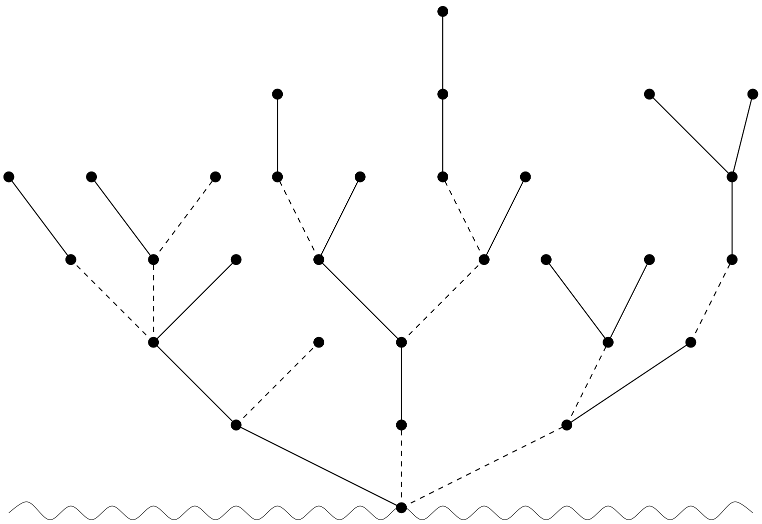
for any games x and y.

Problem 4

Show that the game $\{0|*\}$ is positive but less than any positive number!

Problem 5

Compute the value of the following Blue-Red Hackenbush position. Who will win the game?



Problem 6

Let $G = \{\,\{7|5\}, \{10\,||\,5|3\}\ |\ \{1|0\} - \{7|0\}\,\}$.

(a) Draw the thermograph of G.
(b) What is the temperature of G?
(c) What is the mean value of G?
(d) Who will win G?
(e) Who will win the game $7G - 20$?

Problem 7

Write the following short games on canonical form:

(a) $*2$,
(b) $\{\,\{*|0\},\ 0\ ||\ \{*|0\}\,|*\,\}$.

Problem 8

Find two different short games $G \neq H$ which have the same thermograph, that is, for any number x and any nonnegative real number t, we have $x \leq G_t \Leftrightarrow x \leq H_t$ and $x \geq G_t \Leftrightarrow x \geq H_t$.