SF2972 GAME THEORY
Problem set 1

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1. Peters (2015) Problem 2.5.
2. Peters (2015) Problem 3.3.
3. Peters (2015) Problem 3.4.
4. Consider the 2-player normal-form game:

	A	B
A	a, b	$0, 0$
B	$0, 0$	$1, 1$

   for arbitrary payoffs $a, b > 0$.

   (a) Draw a diagram showing each player's set of best replies to any mixed-strategy profile. [Suggestion: let the horizontal axis be 1's probability (x) for playing her first pure strategy, and let the vertical axis be 2's probability (y) for playing his first pure strategy.]

   (b) Use the diagram to identify the set of Nash equilibria.

   (c) Study how the mixed Nash equilibrium depends on a and b, in particular if 1's probability depends on her payoff a or on the other player's payoff b. Explain!

   (d) Calculate the (expected) payoffs in the mixed equilibrium and compare with the payoffs in the two strict equilibria.

   (e) Show that each player's mixed Nash equilibrium strategy also is the player's min-max strategy against the other player.

5. Consider a two-player simultaneous-move game G with normal form

	L	M	R
A	$8, 11$	$-3, 0$	$0, 0$
B	$9, -1$	$4, 1$	$0, 0$
C	$0, -2$	$0, 0$	$1, 4$

   (a) Find all *rationalizable* pure strategies in G.

   (b) Find all Nash equilibria (in pure and mixed strategies).

   (c) Find all (normal-form) *perfect equilibria* (in pure and mixed strategies).

   (d) Find all *proper equilibria* (in pure or mixed strategies).

6. Consider the following normal-form game G, arising from price competition between two firms with the same average and marginal cost $c = 1$ per unit facing aggregate demand $D(p) = 10 - 2p$. (Hence, the monopoly price in this market is $p = 3$ and the monopoly profit is 8.) Each firm i is constrained to choose an integer price, $p_i \in P = \{0, 1, 2, 3, 4\}$. In the following payoff bi-matrix, firm 1 chooses row and firm two column:

$p_1 \backslash p_2$	0	1	2	3	4
0	$-5, -5$	$-10, 0$	$-10, 0$	$-10, 0$	$-10, 0$
1	$0, -10$	$0, 0$	$0, 0$	$0, 0$	$0, 0$
2	$0, -10$	$0, 0$	$3, 3$	$6, 0$	$6, 0$
3	$0, -10$	$0, 0$	$0, 6$	$4, 4$	$8, 0$
4	$0, -10$	$0, 0$	$0, 6$	$0, 8$	$3, 3$

   (a) Find all strictly dominated pure strategies.

   (b) Find all pure-strategy Nash equilibria.

   (c) Find all weakly dominated strategies.

   (d) Find all perfect pure-strategy Nash equilibria.

7. Lecture Notes Example 12.

8. The example at the end of Sethi and Weibull (2016).