Report 2 Instructions - SF2980 Risk Management

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Instructions

Objectives

The projects are intended as open ended exercises suitable for deeper investigation of some selected topics. Students will

- practice by applying the theory and methods on real examples
- develop their own statistical software functions in programs such as "R" or "MatLab" to solve the problems
- make qualitative and quantitative statements and conclusions about the risk management problems studied in the projects
- present the problems, relevant theory, results and conclusions in written reports
- present the problems, relevant theory, results and conclusions in oral presentations and organized discussions.

Format

- Students will work on the projects in groups of at most four people.
- The projects will be presented and evaluated in written form by handing in two written reports.
- The projects will be presented orally at two presentation seminars.

Grading

On each report you will be given a score in the range 0-25; the total score for the two reports is at most 50. A combined score of at least **25 pts is needed to pass** the assignments part of the course. Grades will be based on the following criteria.

- **Objectives.** A clear description of the project and its objectives.
- **Mathematical background.** A clear and concise presentation of the relevant mathematical background.
- **Results.** A clear and concise presentation of your solution and results. You may also add your own explorations/extensions that you find relevant.
- **Summary.** A clear and consider summary of your results.

Report Template

To get started with the report there is a report template in LaTeX available for download on the course webpage. Please use the sections outlined in the report template. If you are not able to use LaTeX to typeset your report you may use other software (e.g. Word, Pages, etc.)

KTH Finance Lab

Part (c) in the scenario-based risk analysis project will be done, partly, with KTH Finance-Lab. FincanceLab will be used to incorporate real data into the assignment, but all coding can be done in 'R' or Matlab. You can log in to KTH FinanceLab at

`http://www.math.kth.se/matstat/finance/financelab/index.html`

If you experience problems using FinanceLab you may get support by Alexander Aurell `aaurell@kth.se`.

Report II

In the second report (Report II), due on **December 7 at 3 p.m.**, you will present your analysis and results on the projects below. The report must be handed in on **paper** (no emails), be **typeset on a computer** (no handwriting), and be **stapled** in the top left corner.

Scenario-based risk analysis

Part (a) and (b) are identical to Project 10 pp. 328-329 in the book. Part (c) is new.

Consider a stylized model of a life insurer. The insurer faces a liability cash flow of 100 each year for the next 30 years. The current zero rates are given in Table 1, from which the current value of the liability can be computed. In the market there is a short supply of bonds with maturity longer than 10 years. Therefore the insurer has purchased a bond portfolio with payments only within the next 10 years. The bond portfolio has the cash flow given in Table 1. The insurer has also invested in a stock portfolio. The initial capital of the insurer is 30% more than the current value of the liability. The insurer invests 70% of the initial capital in the bond portfolio and 30% of the initial capital in the stock portfolio. The objective in this project is to identify the most dangerous extreme scenario.

Suppose that there are two risk factors in the model, the log-return Y_1 of the stock portfolio and the size Y_2 of a parallel shift of the zero-rate curve. The risk factors are assumed to have a bivariate Normal distribution, means μ_1, μ_2, standard deviations σ_1, σ_2, and linear correlation coefficient ρ given by

$$\mu_1 = 0.08, \quad \mu_2 = 0, \quad \sigma_1 = 0.2, \quad \sigma_2 = 0.01, \quad \rho = 0.1.$$

Consider equally likely extreme scenarios in the following sense. The risk factors can be represented via two independent standard Normally distributed random variables Z_1 and Z_2 as

$$\begin{aligned} Y_1 &= \mu_1 + \sigma_1 Z_1, \\ Y_2 &= \sigma_2 \Big(\rho Z_1 + \sqrt{1-\rho^2} Z_2 \Big). \end{aligned}$$

All scenarios with $\sqrt{Z_1^2 + Z_2^2} = 3$ can be viewed as equally likely extreme scenarios corresponding to three-standard-deviations movements. The extreme scenarios for Z_1, Z_2 translate into extreme scenarios for the risk factors Y_1, Y_2 by the relation above.

(a) Plot the value of the insurer's portfolio, assets minus liabilities, in one year for all the equally likely extreme scenarios.

(b) Identify which scenario for Y_1, Y_2 leads to the worst outcome for the value of the insurer's assets minus that of the liabilities in one year.

(c) [FinanceLab] In this part you will use FinanceLab and repeat part (b) for a realistic non-life insurance portfolio under the current market conditions using estimated principal components of the zero rate curve. The insurer's liability cash flow consists of quarterly payments for the next 10 years (40 payments in total) and the liability cash flow payments are given in Table 2. With the help of FinanceLab you will obtain a table of zero rates and bond payments, similar to Table 1, for a bond portfolio that is immune to small changes in the zero rate curve in the direction of the first principal component (see Section 3.6.1 in the course book). The objective is to compute the scenario Y_1, Y_2 that leads to the worst outcome for the value of the insurer's assets minus liabilities. In this exercise the time horizon is one quarter (0.25 years) and Y_1 represents the log-return of the stock portfolio and Y_2 represents the change of the zero rate curve in the direction of the first principal component.

In this part you may assume that the value of the insurer's initial capital is 30% higher than the value of the liability and that the insurer purchases the immunization portfolio whose value is equal to the value of the liability and invests the remaining capital in the stock portfolio. You may assume that the parameters are given as follows:

$$\mu_1 = 0.04, \quad \mu_2 = 0, \quad \sigma_1 = 0.2, \quad \sigma_2 = \sqrt{\text{“the largest eigenvalue”}}, \quad \rho = -0.1.$$

Here follows a brief explanation on the different parts of the tabs in the QuantLab workspace that will be used for this project.

Init data. In the left panel, select the interest rate data `c` to be used for estimating the principal components and eigenvalues (use the default: SEK3MSWAP). Select the type of bonds `bonds` to be used in the immunization portfolio (use the default: SEKGOVTBOND) and select which individual bonds to include by adjusting the bond weights to 0 or 1. When completed, click `recalc`.

Show sorted eigenvalues. Displays a vector of the sorted eigenvalues.

Show sorted and reduced eigenvectors. Displays the eigenvectors. You can select how many eigenvectors to include by changing the `pca_dim` in the init data tab (use `pca_dim` 1 for this exercise).

Show bond portfolio flows. Gives the cash flow times for the immunization portfolio, the corresponding bond payments and the corresponding zero rates.

You can export the necessary information from QuantLab by copy/paste to a `.csv` or `.txt` file that you can read into 'R' or Matlab.

Table 1: The table shows the annual cash flow of the bond portfolio and the zero rates.

Time	1	2	3	4	5	6	7	8	9	10
Bond payment	4	2	3	1	4	2	3	1	5	5
Zero rate (%)	2.86	3.24	3.55	3.93	4.27	4.62	4.96	5.30	5.55	5.80
Time	11	12	13	14	15	16	17	18	19	20
Bond payment	0	0	0	0	0	0	0	0	0	0
Zero rate (%)	6.05	6.30	6.45	6.60	6.74	6.90	7.00	7.21	7.32	7.32
Time	21	22	23	24	25	26	27	28	29	30
Bond payment	0	0	0	0	0	0	0	0	0	0
Zero rate (%)	7.40	7.48	7.56	7.64	7.70	7.77	7.83	7.90	7.95	8.00

Table 2: Liability cash flow for the non-life insurer in part (c).

Quarter	1	2	3	4	5	6	7	8	9	10
Cash flow	27.2	49.47	67.7	82.63	67.65	55.39	45.35	37.13	30.4	24.89
Quarter	11	12	13	14	15	16	17	18	19	20
Cash flow	20.38	16.68	13.66	11.18	9.16	7.5	6.14	5.02	4.11	3.37
Quarter	21	22	23	24	25	26	27	28	29	30
Cash flow	2.76	2.26	1.85	1.51	1.24	1.01	0.83	0.68	0.56	0.46
Quarter	31	32	33	34	35	36	37	38	39	40
Cash flow	0.37	0.31	0.25	0.20	0.17	0.14	0.11	0.09	0.08	0.06

Tail dependence in large portfolios

This is identical to Project 11 (a) - (e) pp. 328-329 in the book.

Let $Z_1, \dots, Z_{50}$ represent log-returns from today until tomorrow for 50 hypothetical financial assets. Suppose that Z_k has a Student's t distribution with three degrees of freedom and standard deviation 0.01 for each k and that $\tau(Z_j, Z_k) = 0.4$ for $j \neq k$.

Consider an investment of $20,000$ dollars in long positions in each of the assets. Let V_0 and V_1 be the portfolio value today and tomorrow, respectively. Investigate the effect of tail dependence on the distribution of the portfolio value V_1 tomorrow and the distribution of the portfolio log-return $\log(V_1/V_0)$ by simulating from the distribution of V_1. Simulate from the distribution of V_1 under the assumption that

(a) $(Z_1, \dots, Z_{50})$ has a Gaussian copula,

(b) $(Z_1, \dots, Z_{50})$ has a t_4-copula,

(c) $(Z_1, \dots, Z_{50})$ has a Clayton copula.

(d) How large sample size is needed in order to get stable estimates of $\mathrm{VaR}_{0.01}(V_1 - V_0)$ and $\mathrm{ES}_{0.01}(V_1 - V_0)$? Explain the differences in the estimates of $\mathrm{VaR}_{0.01}(V_1 - V_0)$ and $\mathrm{ES}_{0.01}(V_1 - V_0)$ in the three cases (a)-(c).

(e) Compare the results in (a)-(d) to the results when $1,000,000$ dollars is invested in only one of the assets.