Avd. Matematisk statistik

KTH Matematik

EXAM IN SF2937 RELIABILITY THEORY
SATURDAY OCTOBER 23RD 1300-18.00.

Examinor: Gunnar Englund, tel. 7907416, e-mail gunnare@math.kth.se

Allowed material: Formulas and tables for the basic course. Formulas for Reliability theory. Calculator.

The results will be reported to Ladok at the latest 3 weeks after the exam and will thereafter be available through "Mina sidor". If you want your result by e-mail send an e-mail to gunnare@math.kth.se requesting this.

Notation should be defined. Arguments and calculations should be extensive enough to be understood clearly. Numerical answers should be given with at least 2 decimal accuracy.

The limit for E is 24 points. A follow-up exam (komplettering) can be made if you have 22–23 points.

Uppgift 1

A Markov chain has the states $\{1, 2, 3\}$ and the transistion matrix

$$P = \begin{pmatrix} 0.3 & 0.3 & 0.4 \\ 0.4 & 0.3 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{pmatrix}$$

The chain starts at time 0 in state 1.

a) Determin the expected number of steps until the chain has visited state 3 twice. (6 p)

b) Show that the chain has an asymtotic distribution and calculate it. (4 p)

Uppgift 2

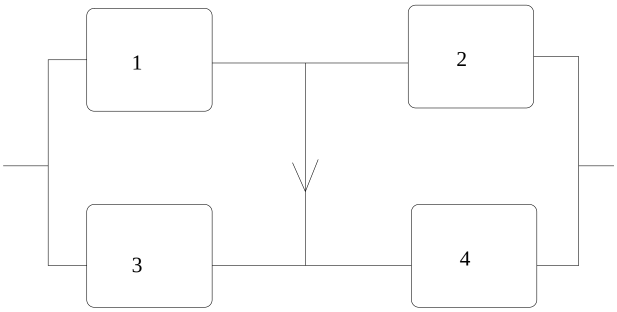
a) A certain type of component has an exponentially distributed life lengths with mean m. To estimate m we tested components in 20 sockets until we obtained 10 failures. Failed components were replaced with new ones. After 15 840 hours we obtained the 10th failure. Calculate a 95% two-sided confidence interval for m. (5 p)

b) Calculate a 95% confidence interval for the probability that a component lasts at least 25 000 hours.

(5 p)

Uppgift 3

I the system below the components work independently of each other and all have the failure rate $\lambda = 2 \cdot 10^{-3} h^{-1}$.



Calculate

a) the structure function in basic form. (2 p)

b) the expected life length. (2 p)

c) the variance of the life length. (3 p)

d) approximately the probability that during 10^5 hours we have had to make more than 250 exchanges when the whole system is exchanged for a new one when it fails. The exchanhe time can be neglected. (3 p)

<u>Hint:</u> A useful formula is

$$E(X^r) = r \cdot \int_0^\infty x^{r-1} (1 - F_X(x))\, dx, \; r > 0$$

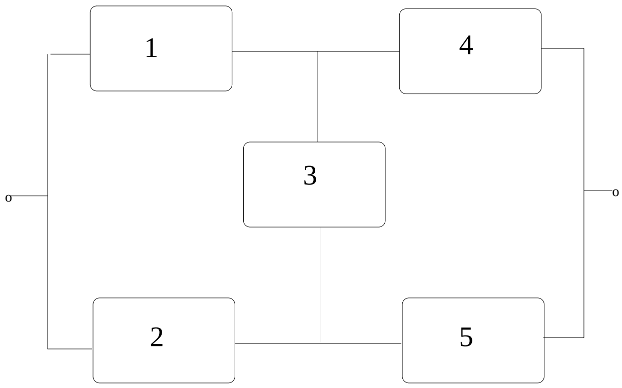
where X is a non-negative random variable with distribution function F_X. Another useful fact is that $\int_0^\infty x^r e^{-x} dx = r!$ if r is integer.

Uppgift 4

A system consists of to components A and B in parallel and they both have failure rate λ. When one of them fails a repair crew is called in- The time until the remair crew arrives is exponentially distributed with expected value $1/\nu$ and then the repair intensity is μ. If another component fails while he is repairing, he starts repairing it when the firts repair is completed.
$\lambda = 10^{-3}$, $\mu = 10 \cdot 10^{-3}$ och $\nu = 20 \cdot 10^{-3} (h^{-1})$

Calculate the asymptotic availability. (10 p)

Uppgift 5



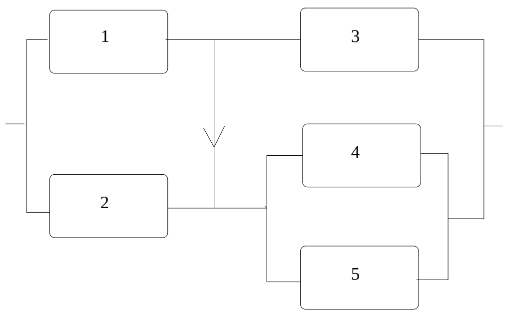
In the system above the life lengths of the components are associated an dhence their state variables are associated at each time. All components have the failure rate λ.

a) Write down the minimal paths and minimal cuts. (2 p)

b) Give an upper and a lower limit for the probability that the sytem works at time t. (5 p)

c) Give an upper and a llower limit for the expected life length of the system. (3 p)

Uppgift 6



a) In the system above the components work independently of each other and have function probabilities

$$p_1 = 0.95 \quad p_2 = 0.9 \quad p_3 = 0.8 \quad p_4 = 0.7 \quad p_5 = 0.75$$

Caclulate the probability that the system works. (4 p)

b) Calculate the Birnbaums measure of component importance for component 1, $I^B(1)$. (3 p)

c) Calculate the Vesely-Fussells measure of importance for component 5, $I^{VF}(5)$. (3 p)