KTH Matematik

EXAMINATION IN SF2975 FINANCIAL DERIVATIVES, 2011-05-27, 08:00–13:00.

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Allowed technical aids: none.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Bonus points: $10-12$ bonus point give full credit for Problem 1. $7-9$ bonus point give full credit for Problem 1a and 1b. $4-6$ bonus points give full credit for Problem 1a.

GOOD LUCK!

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Problem 1

(a) Consider a binomial model with two time steps as in Figure 1. Compute the price of an American put option with maturity 2, strike price 100, on a share of a stock that does not pay dividens. The parameters are $s_0 = 100$, $u = 1.4$, $d = 0.8$, $r = 10\%$, and $p = 0.75$. (4 p)

(b) State and prove the put-call-parity. (3 p)

(c) Consider a Black-Scholes model with a bank account B and a share of a stock S with dynamics

$$\begin{aligned} dB_t &= rB_t dt,\ B_0 = 1, \\ dS_t &= \mu S_t dt + \sigma S_t dW_t,\ S_0 = s. \end{aligned}$$

Suppose the share pays a dividend at time $t_0 \in (0, T)$. The dividend amount is 5% of the share price at t_0. The market is assumed to be free of arbitrage. Derive the dynamics of S under the equivalent martingale measure Q (with B as numeraire). (3 p)

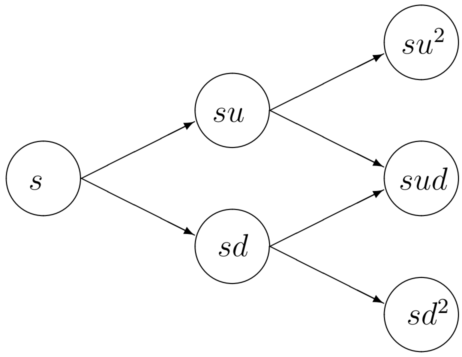

Figure 1: Binomial tree in Problem 1a.

Problem 2

Consider a standard Black-Scholes model with a bank account B and an index of shares S with dynamics (under P)

$$\begin{aligned} dB_t &= rB_t dt,\ B_0 = 1, \\ dS_t &= \mu S_t dt + \sigma S_t dW_t,\ S_0 = s. \end{aligned}$$

where W is a one-dimensional Brownian motion, μ and σ are constants. Suppose the index of shares pays a continuous dividend with dividend yield δ.
Derive an explicit expression for the price of a European option which pays \$100 at time T if $S(T) < K$ and 0 if $S(T) \geq K$. (10 p)

Problem 3

Consider the Ho-Lee model for the short rate $r(t)$, where

$$dr(t) = \Theta(t)dt + \sigma dW^Q(t), \quad r(0) = r_0,$$

and W^Q is a one-dimensional Brownian motion under the equivalent martingale measure Q (with the bank account as numeraire).

Show that the Ho-Lee model has affine term structure and derive an explicit expression for the price, at time $t \geq 0$, of a zero coupon bond with maturity $T > t$. (10 p)

Problem 4

Let $0 = T_0 < T_1 < \cdots < T_n$ and denote by $L_i(t)$ the LIBOR rate at time t for the period T_{i-1} to T_i.

(a) Express the LIBOR rate $L_i(t)$ in terms of prices $p(t, T_{i-1})$ and $p(t, T_i)$ of zero coupon bonds with maturity T_{i-1} and T_i.

(b) Suppose the short rate follows the Ho-Lee model (as defined in Problem 3). Determine the dynamics of the LIBOR rate $L_i(t)$, under the forward measure Q^{T_i}, which is the martingale measure with a zero coupon bond with maturity T_i as numeraire. (10 p)

Problem 5

Consider a standard Black-Scholes model with a bank account B and a share S with dynamics (under Q)

$$\begin{aligned} dB_t &= rB_t dt,\ B_0 = 1, \\ dS_t &= rS_t dt + \sigma S_t dW_t^Q,\ S_0 = s. \end{aligned}$$

where W^Q is a one-dimensional Brownian motion, r and σ are constants. Derive an explicit expression for the price of an Asian option with payoff

$$\max\left\{ \exp\left[\int_0^T \log S(t)dt \right] - K, 0 \right\},$$

at time T. (10 p)