Avd. Matematisk statistik

TENTAMEN I SF2976/5B1576 PORTFÖLJTEORI, FÖRDJUPNINGSKURS ONSDAGEN DEN 19 DECEMBER 2007 KL 08.00–13.00

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Tillåtna hjälpmedel: Inga.

Resultatet anslås senast måndagen den 14 januari 2008.

Införda beteckningar skall förklaras och definieras. Resonemang och uträkningar skall vara så utförliga att de är lätta att följa. Resultat i deluppgift som inte lösts får användas i andra deluppgifter.

Bonus points from the homework and the project can be accounted for in this exam in the following way:
Uppgift 1 will be graded with max(homework grade, points from Uppgift 1 in this exam).

For projects with grades between 0 and 10 points, Uppgift 2 will be graded with max(project grade, points from Uppgift 2 in this exam).

For projects with grades between 11 and 20 points, Uppgift 2 will be graded with 10 points and Uppgift 3 will be graded with max(project grade-10 points, points from Uppgift 3 in this exam).

Gränsen för godkänt är preliminärt 25 poäng.

LYCKA TILL!

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Uppgift 1

Formulate the linear programming algorithm associated with the scenario optimization analysis of a portfolio of N risky assets based the following risk measures:

a) The Conditional Value-at-Risk model. (3 p)

b) The Mean Absolute Deviation. (3 p)

c) Describe how to compute the resampled efficient frontier given a data set with T observations of the N asset returns. (4 p)

Uppgift 2

In the Markowitz framework, derive the formula for optimal weights in a portfolio of assets and liabilities. (10 p)

Uppgift 3

Consider a portfolio of N risky assets with expected returns $\mu = (\mu_1, \dots, \mu_N)$ and covariance matrix $\Sigma = (\sigma_{ij}),\ i, j = 1, \dots N$. Given weights $w = (w_1, \dots, w_N)$, denote the total portfolio's return by $R_p = \sum_{i=1}^{N} w_i R_i$. Assume that $(R_1, \dots, R_N)$ is $N(\mu, \Sigma)$-distributed.

Solve the following optimal asset allocation problem:

$$\begin{aligned}
&\max_w w^t\mu, \quad \text{such that} \\
&P(R_p \leq -L) \leq \alpha, \ \text{for some } \ L > 0, \alpha > 0, \\
&var(R_p) = \sigma_0^2, \\
&\textstyle\sum_{i=1}^N w_i = 1.
\end{aligned}$$

Uppgift 4

Consider a portfolio of N risky assets with expected returns $\pi = (\pi_1, \ldots, \pi_N)$ and covariance matrix $\Sigma = (\sigma_{ij}), \ i, j = 1, \ldots N$, estimated from historical data.
Suppose that you have K uncertain views on future expected returns, μ, for the N assets and you want to use them to modify the expected returns π.

a) State an optimization problem, for an appropriate target function $f(\mu)$, whose optimal solution is the Black-Litterman formula, and solve this problem. Explain carefully the notation you introduce. (5 p)

b) Derive the corresponding covariance matrix given by

$$\Sigma_{predictive} = \left(\frac{1}{2} \frac{\partial^2 f(\mu)}{\partial \mu_i \partial \mu_j} \right)_{ij}$$

evaluated at the optimal solution derived in a). (5 p)

Uppgift 5

Formulate clearly the linear programming algorithm which gives optimal weights of a portfolio of N risky assets obtained by minimizing its total variance given each of the following constraints:

(a) Turnover constraints. (5 p)

(b) Trading constraints. (5 p)