KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2008-12-19, 08:00–13:00.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Good luck!

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Problem 1

Suppose you have access to a large number of historical daily logreturns $x_k, k \geq 1$ of an asset. Each x_k is can be regarded as the outcome of a random variable X_k. You want to use historical logreturns to calculate risk measures using historical simulation. You are concerned that very remote history is not particularly relevant for the near future, but at the same time you don't want to use too short history because it leads to unreliable estimates. You assume that all daily logreturns are independent and identically distributed with a continuous distribution function F. You decide to select n such that the probability that the largest of n logreturns exceeds $\mathrm{VaR}_{0.99}(F)$ with probability at least 0.999. How large does n have to be? (10 p)

Problem 2

Three different samples denoted $X_1^{(1)}, \dots, X_n^{(1)}$, $X_1^{(2)}, \dots, X_n^{(2)}$, and $X_1^{(3)}, \dots, X_n^{(3)}$ are analysed using the Peaks-Over-Threshold (POT) method. For each of the three samples the generalized Pareto distribution GPD with distribution function

$$G_{\gamma,\beta}(x) = 1 - (1 + \gamma x/\beta)^{-1/\gamma}$$

is fitted to the excesses over a threshold u using maximum likelihood estimation. Here u, γ, and β are different for the three samples and we write $\gamma^{(1)}$, $\gamma^{(2)}$, and $\gamma^{(3)}$, respectively.

The three estimated values (from the maximum likelihood estimation) of γ are 0.11, 0.53, and 0.28.

(a) Based on the pairwise QQplots in Figure 1, decide which of the following statements that is true:

(i) $\gamma^{(1)} = 0.11$, $\gamma^{(2)} = 0.53$, $\gamma^{(3)} = 0.28$

(ii) $\gamma^{(1)} = 0.11$, $\gamma^{(2)} = 0.28$, $\gamma^{(3)} = 0.53$

(iii) $\gamma^{(1)} = 0.53$, $\gamma^{(2)} = 0.11$, $\gamma^{(3)} = 0.28$

(iv) $\gamma^{(1)} = 0.53$, $\gamma^{(2)} = 0.28$, $\gamma^{(3)} = 0.11$

(v) $\gamma^{(1)} = 0.28$, $\gamma^{(2)} = 0.53$, $\gamma^{(3)} = 0.11$

(vi) $\gamma^{(1)} = 0.28$, $\gamma^{(2)} = 0.11$, $\gamma^{(3)} = 0.53$

You need to motivate your answer! (5 p)

(b) Determine which of the mean excess plots in Figure 2 (right, middle, left) that is the mean excess plot of sample 1, sample 2, and sample 3. You need to motivate your answer! (5 p)

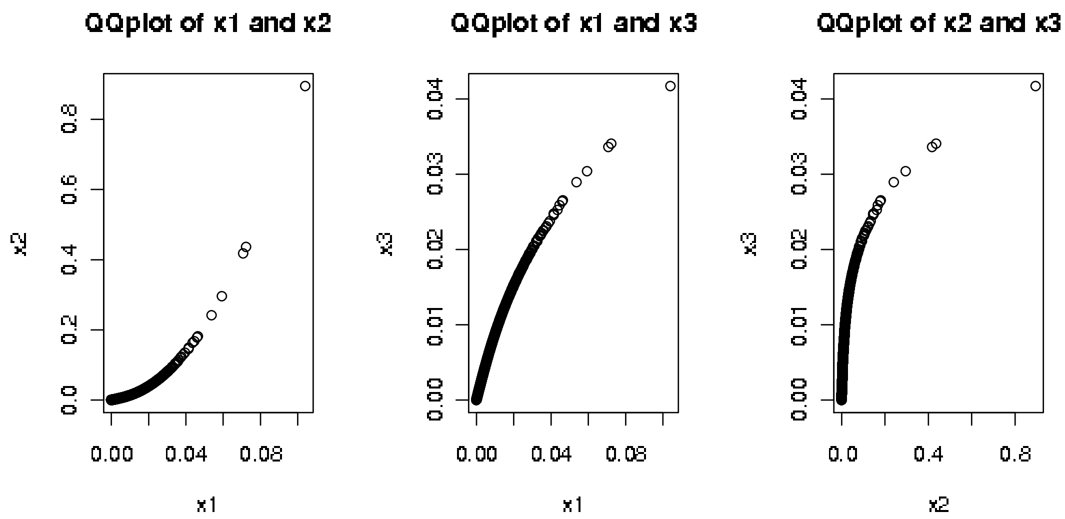

Figure 1: Pairwise qqplots of the three samples

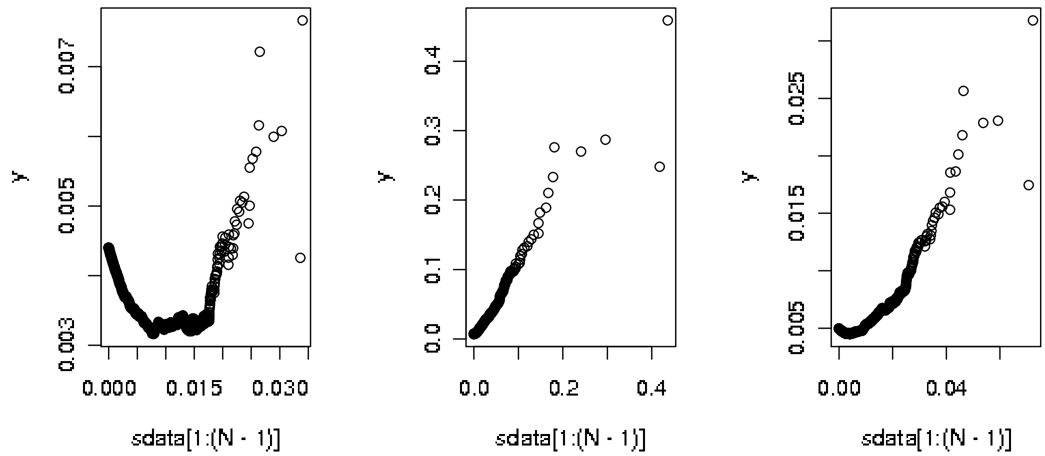

Figure 2: Mean excess plots of the three samples

Problem 3

A scatter plot of sample from the distribution $F(x_1, x_2)$ of a two-dimensional random vector (X_1, X_2) is given in Figure 3. Some QQplots of the marginal distributions are given in Figure 4. Based on this information, do you think F is the distribution function of an elliptical distribution? Motivate your answer. (10 p)

Figure 3: Scatter plot of sample

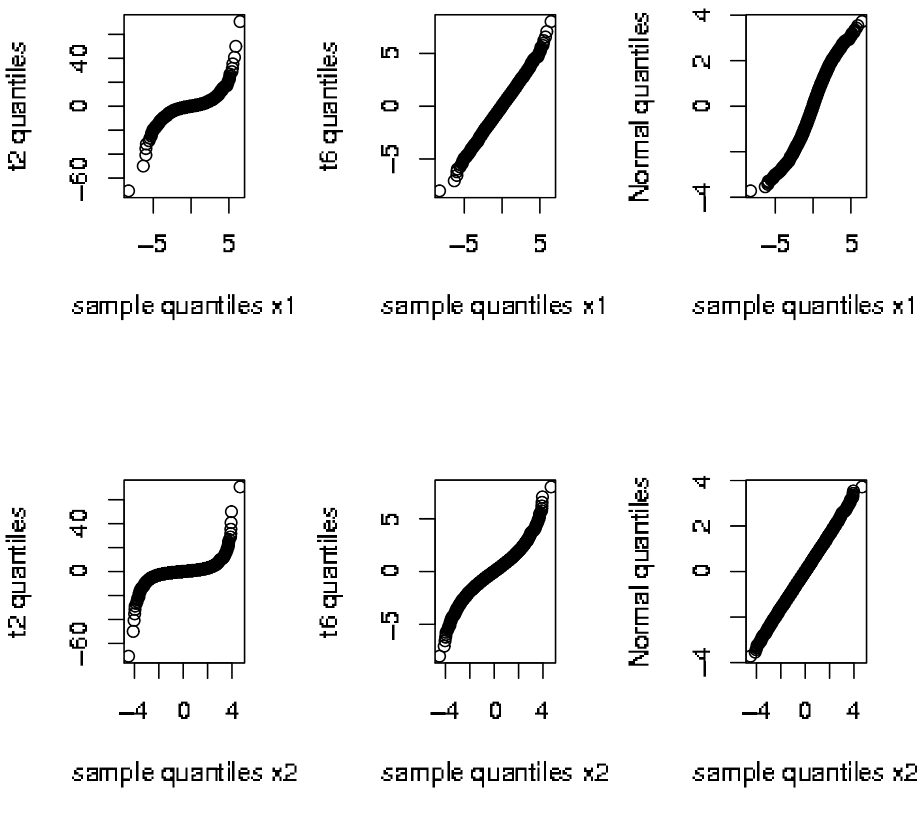

Figure 4: QQplots of the marginal distributions

Problem 4

Suppose you have access to computer software that has a function for generating independent samples from the standard $N(0, 1)$ distribution (one-dimensional). Explain in detail (step-by-step) an algorithm for generating a sample from the random vector (X_1, X_2) with distribution function $F(x_1, x_2)$ of the form $F(x_1, x_2) = C(F_1(x_1), F_2(x_2))$ where $C(u_1, u_2)$ is a Gaussian copula with (correlation) parameter ρ and $F_1(x_1) = 1 - e^{-x_1}$ and $F_2(x_2) = 1 - e^{-2x_2}$. (10 p)

Problem 5

Consider a latent variable model for a portfolio of n credit risks. Let $(Y_1, \ldots, Y_n)$ be the latent variables with continuous marginal distributions $F_1, \ldots, F_n$ and joint distribution function

$$P(Y_1 \leq y_1, \ldots, Y_n \leq y_n) = C(F_1(y_1), \ldots, F_n(y_n))$$

for a copula C. Let $(X_1, \ldots, X_n)$ be the default indicators. For $p \in (0, 1)$ let

$$X_i = 1 \Leftrightarrow Y_i \leq F_i^{\leftarrow}(p).$$

Let $\rho_L(X_i, X_j)$ denote the linear correlation between the default indicators X_i and X_j and $\lambda_L(Y_i, Y_j)$ be the lower tail dependence coefficient between the latent variables Y_i and Y_j. Show that

$$\lim_{p \to 0} \rho_L(X_i, X_j) = \lambda_L(Y_i, Y_j).$$

(10 p)