KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2009-06-08, 14:00–19:00.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

- -

Stock	day01	day02	day03	day04	day05	day06	day07	day08	day09	day10
A	0.001	-0.008	0.002	0.013	0.007	-0.019	0.019	-0.003	-0.002	-0.005

Table 1: Observed daily log-returns for stock A.

Problem 1

Consider a portfolio consisting of a long position of one share of stock A. The stock price today is $S_A = 100$. In Table 1 you find 10 observations of previous daily logreturns. The observations can be considered as outcomes of random variables $X_1, \dots, X_{10}$ that are assumed to be iid.

(a) Use historical simulation to compute the empirical estimate of $\mathrm{VaR}_{0.90}(L^{\Delta})$, where L^{Δ} denotes the linearized portfolio loss over the time period today-until-tomorrow. (5 p)

(b) A colleague suggests a confidence interval for $\mathrm{VaR}_{0.90}(L^{\Delta})$ given by

$$(100 Z_{4:10}, 100 Z_{1:10}),$$

where $Z_{1:10} \geq Z_{2:10} \geq \cdots \geq Z_{10:10}$ is the ordered sample based on $Z_1, \dots, Z_{10}$ and $Z_i = -X_i$, $i = 1, \dots, 10$. Calculate the exact confidence level of this interval. (5 p)

HINT: A table of the Binomial distribution is given at the end of the exam.

Problem 2

Suppose today's value of a portfolio is 100. A qqplot of historical daily logreturns of the portfolio against a standard normal distribution is given in Figure 1. Based on the qqplot, determine an estimate of the linearized one-day Value-at-Risk for the portfolio at level 95%. (10 p)

HINT: A table of the standard normal quantiles is given at the end of the exam.

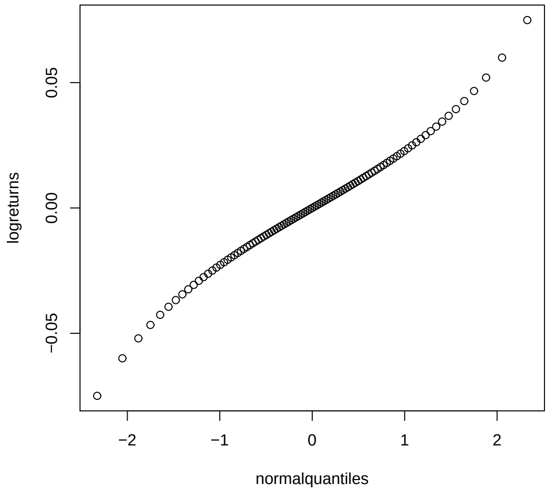

Figure 1: QQplot of the portfolio's daily logreturns and standard normal quantiles.

Problem 3

A portfolio has exposure to two different stocks, called stock A and stock B. Today's value of one share of stock A is $S_A = 50$ and one share of stock B is worth $S_B = 50$. Pairwise historical daily logreturns over the last 1000 days of the two stocks is illustrated in Figure 2.

Calculate an estimate of the linearized one-day Value-at-Risk at level 0.999 for the following portfolios.

(a) A portfolio consisting of a long position of one share in stock A and a long position of one share in stock B. (4 p)

(b) A portfolio consisting of a short position of one share in stock A and a short position of one share in stock B. (4 p)

(c) Explain why there is a significant difference in (a) and (b). (2 p)

Problem 4

Suppose the random vector (X_1, X_2) has distribution function $F(x_1, x_2)$ of the form $F(x_1, x_2) = C(\Phi(x_1), \Phi(x_2))$ where $C(u_1, u_2)$ is a copula and Φ is the standard normal distribution function. For each the following statements, determine if it is *true* or *false*. Motivate your answer!

(a) If C is a Gaussian copula with correlation parameter $\rho = 0$, then F is a spherical distribution. (5 p)

(b) If C is a t_ν copula with $\nu = 4$ and correlation parameter $\rho = 0.5$, then F is an elliptical distribution. (5 p)

Problem 5

Does default correlation increase risk? Consider a portfolio of n credit risks. Let $X_1, \ldots, X_n$ be the default indicators and assume $EX_i = p$ for each $i = 1, \ldots, n$, where $p \in (0, 1)$. Let $\rho_L(X_i, X_j)$ denote the linear correlation between the default indicators X_i and X_j and assume $\rho_L(X_i, X_j) = \rho$ for each $i \neq j$. Let $N = \sum_{i=1}^n X_i$ be the number of defaults. Suppose the distribution of N is approximated by a normal distribution with mean EN and variance $\mathrm{var}(N)$ and that Value-at-Risk, $\mathrm{VaR}_\alpha(N)$, is calculated using this normal approximation. In this case, is $\mathrm{VaR}_\alpha(N)$ increasing? or decreasing? as a function of ρ. Motivate your answer!

(10 p)

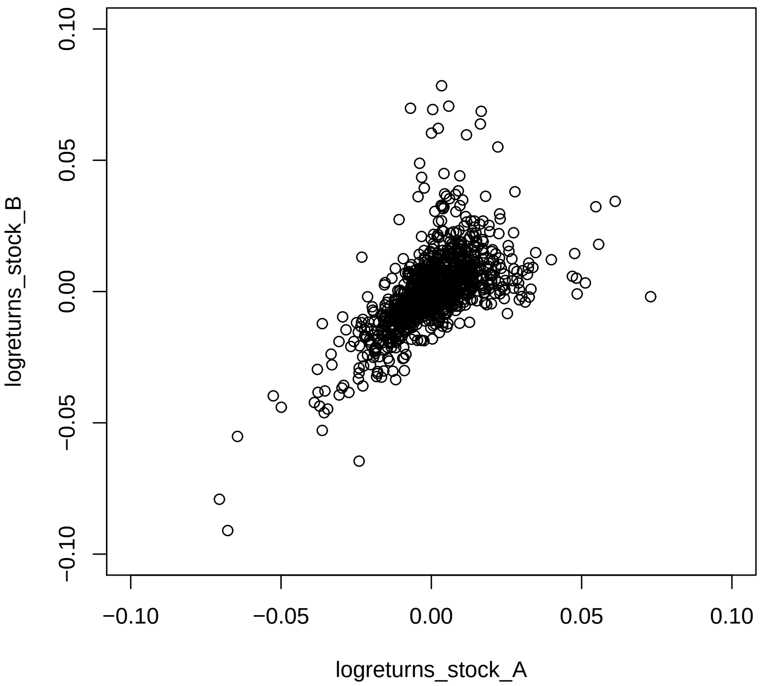

Figure 2: Historical logreturns for stock A and stock B over the last 1000 days.

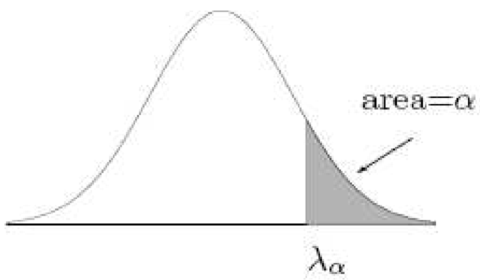

Figure 3: Standard normal quantiles $P(X > \lambda_\alpha) = \alpha$ where $X \sim N(0, 1)$.

α	λ_α
0.1	1.2816
0.05	1.6449
0.025	1.9600
0.01	2.3263
0.005	2.5758
0.001	3.0902
0.0005	3.2905
0.0001	3.7190

Tables of the Binomial distribution, $P(X \leq x)$ where $X \sim Bin(n, p)$.

n	x | p	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
2	0	.90250	.81000	.72250	.64000	.56250	.49000	.36000	.25000
	1	.99750	.99000	.97750	.96000	.93750	.91000	.84000	.75000
3	0	.85737	.72900	.61412	.51200	.42188	.34300	.21600	.12500
	1	.99275	.97200	.93925	.89600	.84375	.78400	.64800	.50000
	2	.99987	.99900	.99662	.99200	.98438	.97300	.93600	.87500
4	0	.81451	.65610	.52201	.40960	.31641	.24010	.12960	.06250
	1	.98598	.94770	.89048	.81920	.73828	.65170	.47520	.31250
	2	.99952	.99630	.98802	.97280	.94922	.91630	.82080	.68750
	3	.99999	.99990	.99949	.99840	.99609	.99190	.97440	.93750
5	0	.77378	.59049	.44371	.32768	.23730	.16807	.07776	.03125
	1	.97741	.91854	.83521	.73728	.63281	.52822	.33696	.18750
	2	.99884	.99144	.97339	.94208	.89648	.83692	.68256	.50000
	3	.99997	.99954	.99777	.99328	.98438	.96922	.91296	.81250
	4	1.00000	.99999	.99992	.99968	.99902	.99757	.98976	.96875
6	0	.73509	.53144	.37715	.26214	.17798	.11765	.04666	.01562
	1	.96723	.88574	.77648	.65536	.53394	.42017	.23328	.10938
	2	.99777	.98415	.95266	.90112	.83057	.74431	.54432	.34375
	3	.99991	.99873	.99411	.98304	.96240	.92953	.82080	.65625
	4	1.00000	.99995	.99960	.99840	.99536	.98906	.95904	.89063
	5	1.00000	1.00000	.99999	.99994	.99976	.99927	.99590	.98438
7	0	.69834	.47830	.32058	.20972	.13348	.08235	.02799	.00781
	1	.95562	.85031	.71658	.57672	.44495	.32942	.15863	.06250
	2	.99624	.97431	.92623	.85197	.75641	.64707	.41990	.22656
	3	.99981	.99727	.98790	.96666	.92944	.87396	.71021	.50000
	4	.99999	.99982	.99878	.99533	.98712	.97120	.90374	.77344
	5	1.00000	.99999	.99993	.99963	.99866	.99621	.98116	.93750
	6	1.00000	1.00000	1.00000	.99999	.99994	.99978	.99836	.99219
8	0	.66342	.43047	.27249	.16777	.10011	.05765	.01680	.00391
	1	.94276	.81310	.65718	.50332	.36708	.25530	.10638	.03516
	2	.99421	.96191	.89479	.79692	.67854	.55177	.31539	.14453
	3	.99963	.99498	.97865	.94372	.88618	.80590	.59409	.36328
	4	.99998	.99957	.99715	.98959	.97270	.94203	.82633	.63672
	5	1.00000	.99998	.99976	.99877	.99577	.98871	.95019	.85547
	6	1.00000	1.00000	.99999	.99992	.99962	.99871	.99148	.96484
	7	1.00000	1.00000	1.00000	1.00000	.99998	.99993	.99934	.99609
9	0	.63025	.38742	.23162	.13422	.07508	.04035	.01008	.00195
	1	.92879	.77484	.59948	.43621	.30034	.19600	.07054	.01953
	2	.99164	.94703	.85915	.73820	.60068	.46283	.23179	.08984
	3	.99936	.99167	.96607	.91436	.83427	.72966	.48261	.25391
	4	.99997	.99911	.99437	.98042	.95107	.90119	.73343	.50000
	5	1.00000	.99994	.99937	.99693	.99001	.97471	.90065	.74609
	6	1.00000	1.00000	.99995	.99969	.99866	.99571	.97497	.91016
	7	1.00000	1.00000	1.00000	.99998	.99989	.99957	.99620	.98047
	8	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99974	.99805

n	x | p	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
10	0	.59874	.34868	.19687	.10737	.05631	.02825	.00605	.00098
	1	.91386	.73610	.54430	.37581	.24403	.14931	.04636	.01074
	2	.98850	.92981	.82020	.67780	.52559	.38278	.16729	.05469
	3	.99897	.98720	.95003	.87913	.77588	.64961	.38228	.17188
	4	.99994	.99837	.99013	.96721	.92187	.84973	.63310	.37695
	5	1.00000	.99985	.99862	.99363	.98027	.95265	.83376	.62305
	6	1.00000	.99999	.99987	.99914	.99649	.98941	.94524	.82813
	7	1.00000	1.00000	.99999	.99992	.99958	.99841	.98771	.94531
	8	1.00000	1.00000	1.00000	1.00000	.99997	.99986	.99832	.98926
	9	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99990	.99902
11	0	.56880	.31381	.16734	.08590	.04224	.01977	.00363	.00049
	1	.89811	.69736	.49219	.32212	.19710	.11299	.03023	.00586
	2	.98476	.91044	.77881	.61740	.45520	.31274	.11892	.03271
	3	.99845	.98147	.93056	.83886	.71330	.56956	.29628	.11328
	4	.99989	.99725	.98411	.94959	.88537	.78970	.53277	.27441
	5	.99999	.99970	.99734	.98835	.96567	.92178	.75350	.50000
	6	1.00000	.99998	.99968	.99803	.99244	.97838	.90065	.72559
	7	1.00000	1.00000	.99997	.99976	.99881	.99571	.97072	.88672
	8	1.00000	1.00000	1.00000	.99998	.99987	.99942	.99408	.96729
	9	1.00000	1.00000	1.00000	1.00000	.99999	.99995	.99927	.99414
	10	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99996	.99951
12	0	.54036	.28243	.14224	.06872	.03168	.01384	.00218	.00024
	1	.88164	.65900	.44346	.27488	.15838	.08503	.01959	.00317
	2	.98043	.88913	.73582	.55835	.39068	.25282	.08344	.01929
	3	.99776	.97436	.90779	.79457	.64878	.49252	.22534	.07300
	4	.99982	.99567	.97608	.92744	.84236	.72366	.43818	.19385
	5	.99999	.99946	.99536	.98059	.94560	.88215	.66521	.38721
	6	1.00000	.99995	.99933	.99610	.98575	.96140	.84179	.61279
	7	1.00000	1.00000	.99993	.99942	.99722	.99051	.94269	.80615
	8	1.00000	1.00000	.99999	.99994	.99961	.99831	.98473	.92700
	9	1.00000	1.00000	1.00000	1.00000	.99996	.99979	.99719	.98071
	10	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99968	.99683
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998	.99976
13	0	.51334	.25419	.12091	.05498	.02376	.00969	.00131	.00012
	1	.86458	.62134	.39828	.23365	.12671	.06367	.01263	.00171
	2	.97549	.86612	.69196	.50165	.33260	.20248	.05790	.01123
	3	.99690	.96584	.88200	.74732	.58425	.42061	.16858	.04614
	4	.99971	.99354	.96584	.90087	.79396	.65431	.35304	.13342
	5	.99998	.99908	.99247	.96996	.91979	.83460	.57440	.29053
	6	1.00000	.99990	.99873	.99300	.97571	.93762	.77116	.50000
	7	1.00000	.99999	.99984	.99875	.99435	.98178	.90233	.70947
	8	1.00000	1.00000	.99998	.99983	.99901	.99597	.96792	.86658
	9	1.00000	1.00000	1.00000	.99998	.99987	.99935	.99221	.95386
	10	1.00000	1.00000	1.00000	1.00000	.99999	.99993	.99868	.98877
	11	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99986	.99829
	12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99988

n	x | p	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50
14	0	.48767	.22877	.10277	.04398	.01782	.00678	.00078	.00006
	1	.84701	.58463	.35667	.19791	.10097	.04748	.00810	.00092
	2	.96995	.84164	.64791	.44805	.28113	.16084	.03979	.00647
	3	.99583	.95587	.85349	.69819	.52134	.35517	.12431	.02869
	4	.99957	.99077	.95326	.87016	.74153	.58420	.27926	.08978
	5	.99997	.99853	.98847	.95615	.88833	.78052	.48585	.21198
	6	1.00000	.99982	.99779	.98839	.96173	.90672	.69245	.39526
	7	1.00000	.99998	.99967	.99760	.98969	.96853	.84986	.60474
	8	1.00000	1.00000	.99996	.99962	.99785	.99171	.94168	.78802
	9	1.00000	1.00000	1.00000	.99995	.99966	.99833	.98249	.91022
	10	1.00000	1.00000	1.00000	1.00000	.99996	.99975	.99609	.97131
	11	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99939	.99353
	12	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99994	.99908
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99994
15	0	.46329	.20589	.08735	.03518	.01336	.00475	.00047	.00003
	1	.82905	.54904	.31859	.16713	.08018	.03527	.00517	.00049
	2	.96380	.81594	.60423	.39802	.23609	.12683	.02711	.00369
	3	.99453	.94444	.82266	.64816	.46129	.29687	.09050	.01758
	4	.99939	.98728	.93829	.83577	.68649	.51549	.21728	.05923
	5	.99995	.99775	.98319	.93895	.85163	.72162	.40322	.15088
	6	1.00000	.99969	.99639	.98194	.94338	.86886	.60981	.30362
	7	1.00000	.99997	.99939	.99576	.98270	.94999	.78690	.50000
	8	1.00000	1.00000	.99992	.99922	.99581	.98476	.90495	.69638
	9	1.00000	1.00000	.99999	.99989	.99921	.99635	.96617	.84912
	10	1.00000	1.00000	1.00000	.99999	.99988	.99933	.99065	.94077
	11	1.00000	1.00000	1.00000	1.00000	.99999	.99991	.99807	.98242
	12	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99972	.99631
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99951
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99997
16	0	.44013	.18530	.07425	.02815	.01002	.00332	.00028	.00002
	1	.81076	.51473	.28390	.14074	.06348	.02611	.00329	.00026
	2	.95706	.78925	.56138	.35184	.19711	.09936	.01834	.00209
	3	.99300	.93159	.78989	.59813	.40499	.24586	.06515	.01064
	4	.99914	.98300	.92095	.79825	.63019	.44990	.16657	.03841
	5	.99992	.99670	.97646	.91831	.81035	.65978	.32884	.10506
	6	.99999	.99950	.99441	.97334	.92044	.82469	.52717	.22725
	7	1.00000	.99994	.99894	.99300	.97287	.92565	.71606	.40181
	8	1.00000	.99999	.99984	.99852	.99253	.97433	.85773	.59819
	9	1.00000	1.00000	.99998	.99975	.99836	.99287	.94168	.77275
	10	1.00000	1.00000	1.00000	.99997	.99971	.99843	.98086	.89494
	11	1.00000	1.00000	1.00000	1.00000	.99996	.99973	.99510	.96159
	12	1.00000	1.00000	1.00000	1.00000	1.00000	.99997	.99906	.98936
	13	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99987	.99791
	14	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99999	.99974
	15	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	.99998

SOLUTION TO EXAMINATION IN SF2980 RISK MANAGEMENT 2008-06-04.

Problem 1

(a) The historical simulation estimate of $\mathrm{VaR}_{0.9}(L^\Delta)$ is $L^\Delta_{[10(1-0.9)]+1:10} = L^\Delta_{2:10}$ where $L^\Delta_i = -S_A X_i = 100 Z_i$. That is, the estimate is $\hat{q}_{0.9} = 100 Z_{2,10} = 0.8$.
(b) The interval can miss the true value $q_{0.9}$ either to the left (too few large losses) or to the right (too many large losses). The probability that it misses to the left is $P(Z_{1,10} \le q_{0.9}) = P(Bin(10, 0.1) = 0) = 0.345$ and the probability that it misses to the right is $P(Z_{4,10} > q_{0.9}) = 1 - P(Bin(10, 0.1) \le 3) = 0.013$. So confidence level is $1 - 0.345 - 0.013 = 0.642$.

Problem 2

The qqplot plots the standard normal quantiles (x-axis) versus the empirical quantiles of the portfolio logreturns (y-axis). The points of the qqplot is of the form $(\Phi^{-1}(p_i), F_n^{\leftarrow}(p_i))$ for a number of different $p_i \in (0,1)$. We will assume the empirical quantile $F_n^{\leftarrow}(p_i)$ is close to the true quantile of portfolio logreturns. If X denotes a daily portfolio logreturn, then the linearized one day loss is given by $L^\Delta = -100X$ and $\mathrm{VaR}_{0.95}(L^\Delta) = 100\,\mathrm{VaR}_{0.95}(-X)$. To find $\mathrm{VaR}_{0.95}(-X)$ we see from the table that the standard normal 95%-quantile is 1.63. Thus, $\mathrm{VaR}_{0.95}(-X)$ is the y-value in the qqplot that corresponds to -1.63 on the x-axis. That is, $\mathrm{VaR}_{0.95}(-X) \approx 0.045$ and $\mathrm{VaR}_{0.95}(L^\Delta) \approx 4.5$.

Problem 3

(a) Here $L^\Delta = -50(X_A + X_B)$ where X_A and X_B are the daily logreturns. We see that large negative values of $X_A + X_B$ lead to large losses. Since $[1000(1-0.999)]+1 = 2$ the empirical estimate is $L^\Delta_{2:1000}$. Looking in the lower left corner of the sample we see that $L^\Delta_{2:1000} \approx -50(-0.07 - 0.075) = 7.25$.
(b) Here $L^\Delta = 50(X_A + X_B)$ where X_A and X_B are the daily logreturns. We see that large positive values of $X_A + X_B$ lead to large losses. Since $[1000(1-0.999)]+1 = 2$ the empirical estimate is $L^\Delta_{2:1000}$. Looking in the upper right corner of the sample it is a difficult by eyesight to determine exactly which point that corresponds to $L^\Delta_{2:1000}$. However, all candidates will give approximately the same value for the sum $X_A + X_B$. We selected $L^\Delta_{2:1000} \approx 50(0.055 + 0.04) = 4.25$.
(c) The difference is large because the dependence is strong in the lower tail (where X_A and X_B are negative). The plot indicate that there may be a strong lower tail dependence. In contrast the dependence is weak in the upper tail (where X_A and X_B are positive). The plot indicate weak or zero upper tail dependence.

Problem 4

(a) True: F is the distribution function of a standard normal distribution which is spherical. (b) False, if the distribution is elliptical then the marginal distributions and the copula must be elliptical with the same generator (same distribution of R).

Problem 5

The variance of N can be calculated as

$$\begin{aligned}
\mathrm{var}(N) = EN^2 - (EN)^2 &= E\Big(\sum_{i=1}^{n}\sum_{j=1}^{n} X_iX_j\Big) - (np)^2 \\
&= \sum_{i=1}^{n} EX_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} E(X_iX_j) - (np)^2 \\
&= np + n(n-1)[\rho p(1-p) + p^2] - (np)^2.
\end{aligned}$$

This is increasing in ρ. By the normal approximation we have

$$\mathrm{VaR}_\alpha(N) \approx EN + \sqrt{\mathrm{var}(N)}\Phi^{-1}(\alpha),$$

which is increasing in $\mathrm{var}(N)$ and hence increasing in ρ.