KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2014-01-14, 14:00–19:00.

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*Allowed technical aids*: Everything except computers and communication devices. All books, notes, and similar are allowed. A calculator is necessary.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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## Problem 1

An investor invests 100 SEK in a long position of a share of a Swedish stock. A quantile-quantile-plot of the historical daily log-returns of the share price against a standard normal distribution is displayed in Figure 1. Compute the Value-at-Risk at level 0.01 of the net value of the investment over one day. A table of the normal distribution is given at the end of the exam. (10 p)

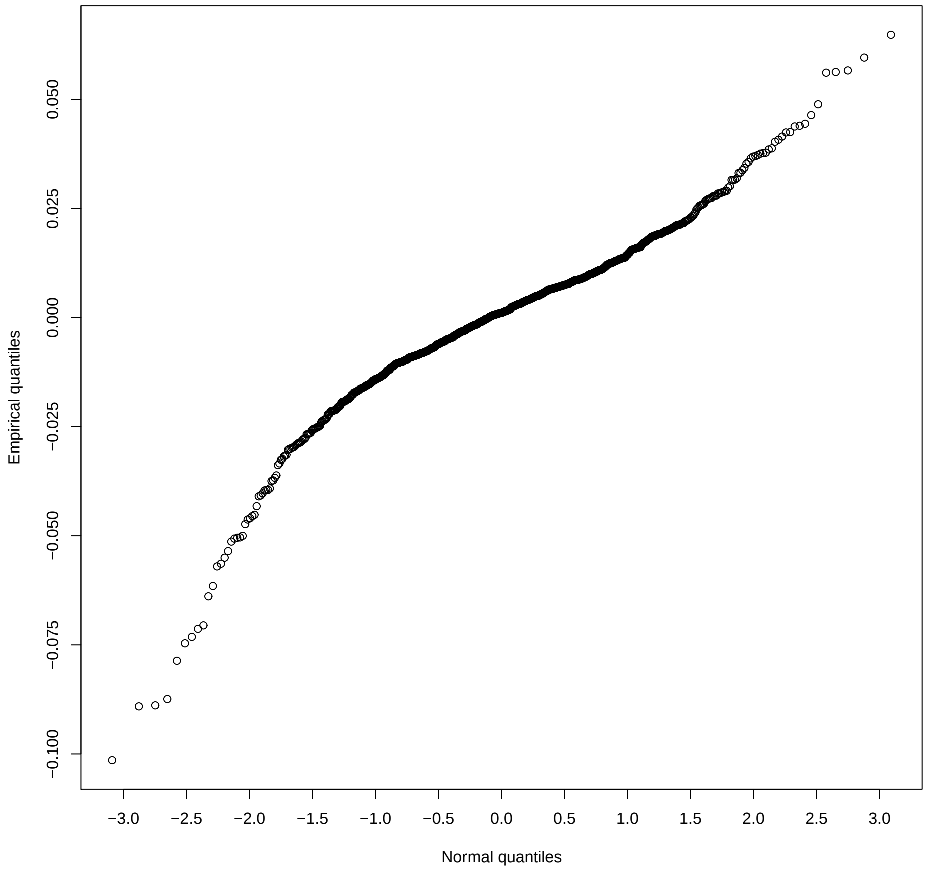
## Problem 2

Let $I_1$ and $I_2$ be default indicators of two firms and suppose that the default probability is $p$. That is, $P(I_1 = 1) = P(I_2 = 1) = p \in (0, 1)$.

(a) Suppose $(I_1, I_2)$ is modelled by a latent variable model, where $I_k = I\{Y_k \leq d_k\}$, $k = 1, 2$, and the latent variables $(Y_1, Y_2)$ has a Clayton copula with parameter $\theta$ and standard normal marginal distributions. Determine the default correlation $\mathrm{Cor}(I_1, I_2)$ (standard linear correlation). (7 p)

(b) Let $(J_1, J_2)$ be default indicators modelled by the Beta mixture model. That is, $J_1$ and $J_2$ are conditionally independent and $\mathrm{Be}(z)$-distributed given $Z = z$, where $Z$ has a $\mathrm{Beta}(a, b)$ distribution. Determine the parameters $a$ and $b$ (expressed in $p$ and $\theta$) such that $P(J_1 = 1) = P(J_2 = 1) = p$ and $\mathrm{Cor}(J_1, J_2) = \mathrm{Cor}(I_1, I_2)$. (3 p)

Figure 1: Quantile-quantile plot of daily log-returns against standard normal distribution




## Problem 3

An equity long-short strategy is an investing strategy, used primarily by hedge funds, that involves taking long positions in stocks that are expected to increase in value and short positions in stocks that are expected to decrease in value. Let's say that a hedge fund believes that Ericsson will perform better than Nokia over the next week. The hedge fund invests 1 million SEK in long positions of Ericsson shares and 1 million SEK in short positions of Nokia shares. Suppose the vector $\mathbf{Y} = (Y_1, Y_2)^T$ of the weekly log-returns of the share prices of Ericsson, $Y_1$, and Nokia, $Y_2$, has an elliptical distribution with representation

$$\mathbf{Y} = \boldsymbol{\mu} + A\mathbf{Z},$$

where $\mathbf{Z}$ has spherical distribution and

$$A = \begin{pmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}.$$

The parameter values are

$$\boldsymbol{\mu} = 10^{-3} \cdot \begin{pmatrix} 2 \\ 1.8 \end{pmatrix}, \quad \sigma_1 = 5.5 \cdot 10^{-3}, \sigma_2 = 4.2 \cdot 10^{-3}, \rho = 0.9.$$

Use *linearization* and determine the most dangerous scenario (the one leading to the smallest portfolio value after one week) among those for which $\mathbf{Z}^T\mathbf{Z} = 25$.

(a) Express the most dangerous scenario in terms of the corresponding outcome for $\mathbf{Y}$. (7 p)

(b) Determine the net profit of the portfolio after one week for the most dangerous scenario. (3 p)

## Problem 4

Consider the investment strategy in Problem 3 and let $\Delta V$ denote the net profit of the strategy over one week. Suppose that $\mathbf{Y}$ has a $t_\nu(\boldsymbol{\mu}, \Sigma)$-distribution with $\nu = 3$ and $\Sigma = AA^T$ and parameter values as in Problem 3.

(a) Determine the level $p$ such that the Value-at-Risk of $\Delta V$ at level $p$ is equal to 10 000. You may use linearization. (8 p)

(b) Determine the level $p$ such that the Value-at-Risk of $\Delta V$ at level $p$ is equal to the loss of the most dangerous scenario in Problem 3(b). You may use linearization. (2 p)

A table of quantiles of the standard $t_3$-distribution is given below.

Table 1: Distribution function of the standard $t_3$ distribution

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 |
| $t_3(x)$ | 0.930 | 0.936 | 0.942 | 0.947 | 0.952 | 0.956 | 0.960 | 0.963 | 0.966 | 0.969 |
| x | 3.0 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 | 3.8 | 3.9 |
| $t_3(x)$ | 0.971 | 0.973 | 0.975 | 0.977 | 0.979 | 0.980 | 0.982 | 0.983 | 0.984 | 0.985 |
| x | 4.0 | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 |
| $t_3(x)$ | 0.986 | 0.987 | 0.988 | 0.988 | 0.989 | 0.990 | 0.990 | 0.991 | 0.991 | 0.992 |
| x | 5.0 | 5.1 | 5.2 | 5.3 | 5.4 | 5.5 | 5.6 | 5.7 | 5.8 | 5.9 |
| $t_3(x)$ | 0.992 | 0.993 | 0.993 | 0.993 | 0.994 | 0.994 | 0.994 | 0.995 | 0.995 | 0.995 |
| x | 6.0 | 6.1 | 6.2 | 6.3 | 6.4 | 6.5 | 6.6 | 6.7 | 6.8 | 6.9 |
| $t_3(x)$ | 0.995 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.996 | 0.997 | 0.997 | 0.997 |
| x | 7.0 | 7.1 | 7.2 | 7.3 | 7.4 | 7.5 | 7.6 | 7.7 | 7.8 | 7.9 |
| $t_3(x)$ | 0.997 | 0.997 | 0.997 | 0.997 | 0.997 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 |
| x | 8.0 | 8.1 | 8.2 | 8.3 | 8.4 | 8.5 | 8.6 | 8.7 | 8.8 | 8.9 |
| $t_3(x)$ | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.998 | 0.999 |

## Problem 5

Let $Y_1, \ldots, Y_n$ be independent and identically distributed random variables with $P(Y_1 > y) = y^{-\alpha}$, $y \geq 1$, $\alpha > 1$. The $Y$-variables represent different insurance losses occurring over a one year period. A reinsurance company can select between a diversified portfolio where they have to pay the amount

$$X_n = \frac{1}{n} \sum_{k=1}^{n} (Y_k - y_0)_+$$

and a non-diversified portfolio where they have to pay $X_1 = (Y_1 - y_0)_+$, where $y_0$ is a fixed retention level. Use the subexponential property to determine an asymptotic approximation (as $p \to 0$) for the diversification factor

$$\delta_p(n) = \frac{\mathrm{ES}_p(X_n)}{\mathrm{ES}_p(X_1)}.$$

(10 p)

Figure 2: Standard Normal distribution

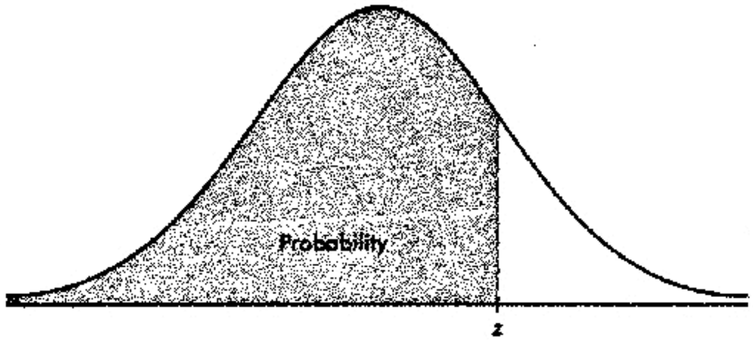



**TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)**

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |