KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2014-03-10, 08:00–13:00.

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Allowed technical aids: Everything except computers and communication devices. All books, notes, and similar are allowed. A calculator is necessary.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

An investor invests 100 SEK in a long position of a share of a Swedish stock that does not pay dividends. A quantile-quantile-plot of 48 historical monthly log-returns of the share price against a standard normal distribution is displayed in Figure 1. Compute the Expected Shortfall at level 0.01 of the net value $(V_{12} - V_0 R_0)$ of the investment over one year. The continuously compounded annual interest rate is assumed to be 3% ($R_0 = e^{0.03}$). A table of the normal distribution is given at the end of the exam.
You are welcome to make well motivated assumptions. All assumptions made must be clearly stated. (10 p)

Problem 2

Suppose $\mathbf{X} = (X_1, X_2)^T$ has an elliptical distribution with dispersion matrix Σ. Four independent samples from the distribution of $\mathbf{X}$ are illustrated in Figure 2. Estimate the linear correlation parameter

$$\varrho = \frac{\Sigma_{12}}{\sqrt{\Sigma_{11}\Sigma_{22}}}.$$

(10 p)

Figure 1: Quantile-quantile plot of monthly log-returns against standard normal distribution (Problem 1).

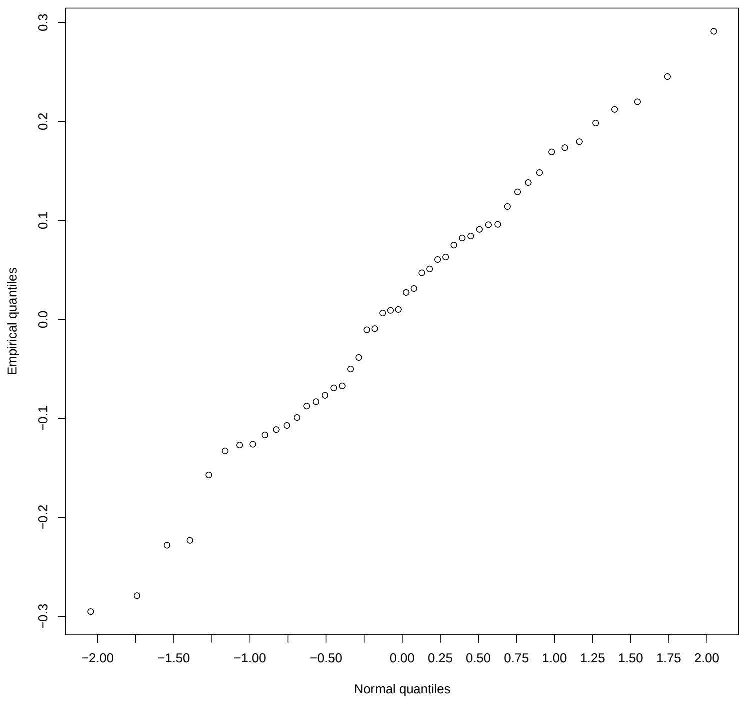
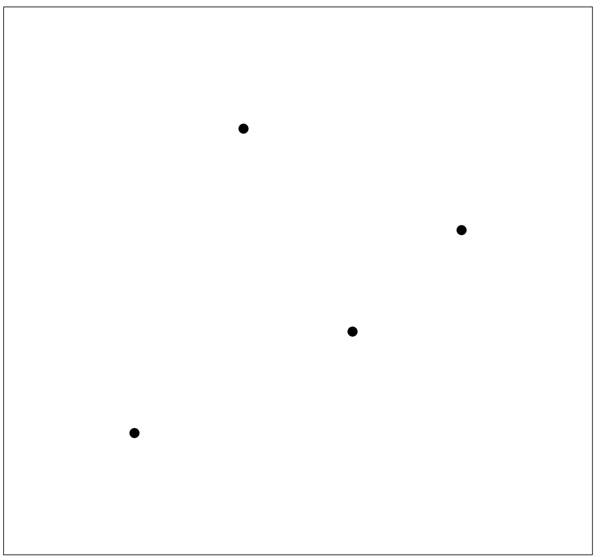


Figure 2: Four independent samples from the distribution of **X** (Problem 2).



Problem 3

Let L_1 and L_2 be independent random variables, representing losses, both with distribution function $F(x) = e^{-\frac{1}{\sqrt{x}}}, x > 0$. Determine which of the portfolios $L_1 + L_2$ and $2L_1$ which is riskiest by determining whether the limit

$$\lim_{x\to\infty} \frac{P(L_1 + L_2 > x)}{P(2L_1 > x)}$$

is greater than 1, smaller than 1, or equal to 1. (10 p)

Problem 4

A trader is betting on changes in implied volatility from time 0 today until time $t > 0$ in the future. Consider two call options on the value of a stock index with maturity in one year and two years respectively. Both options have strike K, at the money, and in particular $S_0 = K = 100$, where S_0 is the current value of the underlying stock index. The trader believes that over a short period of time, from 0 to $t < T_1$ the change in implied volatility for the nearer maturity, $X = \sigma_t^1 - \sigma_0^1$, will be *smaller* than for the more distant maturity, $Y = \sigma_t^2 - \sigma_0^2$. Here $\sigma_0^1 = 0.50$ and $\sigma_0^2 = 0.55$. The trader wants to capitalize in this belief without betting on other potential movements of the underlying stock index.

You may suppose that the interest rate is 0 and that (X, Y) has a joint normal distribution with $E[X] = 0$, $E[Y] = 0.02$, $\mathrm{Var}(X) = 2.5 \cdot 10^{-3}$, $\mathrm{Var}(Y) = 3.0 \cdot 10^{-3}$ and $\mathrm{Cor}(X, Y) = 0.9$. The prices of the call options are given by Black-Scholes formula with the corresponding implied volatilities.

Consider a portfolio with the amount h_0 in cash (on a zero-interest bank account), h_1 units of the underlying stock index, h_2 number of call options with maturity in one year and h_3 number of call options with maturity in two years.

Determine the portfolio h_0, h_1, h_2, h_3, having the desired properties, and such that the portfolio has zero initial value and the Value-at-Risk at level 0.05, for the investment from today until time t, is 10.

You may use linearization. If you cannot compute standard normal densities you may use $\phi(0.025) = 0.4$, where ϕ is the standard normal density. (10 p)

Problem 5

In this problem you will show that a latent variable model with a Clayton copula can be identified with a Bernoulli mixture model.

Let $I_1, \dots, I_n$ be default indicators modelled by a latent variable model in the following way. Let $(Y_1, \dots, Y_n)$ have identically distributed marginal distributions, that is, Y_k has distribution function F for each k, and a Clayton copula C given by

$$C(u_1, \dots, u_n) = (u_1^{-\theta} + \cdots + u_n^{-\theta} + n - 1)^{-1/\theta}.$$

Put $I_k = I\{Y_k \le d\}$ where $d = F^{-1}(p)$ and p is the individual default probability.

A Bernoulli mixture model with mixture variable Z is a model for default indicators $J_1, \dots, J_n$ such that $J_1, \dots, J_n$ are conditionally independent, given Z, with $P(J_k = 1 \mid Z) = Z$.

Determine the distribution of Z such that $(I_1, \dots, I_n)$ and $(J_1, \dots, J_n)$ have the same distribution. That is, for any $(i_1, \dots, i_n)$ with $i_k \in \{0, 1\}$, $k = 1, \dots, n$, it holds that

$$P(I_1 = i_1, \dots, I_n = i_n) = P(J_1 = i_1, \dots, J_n = i_n).$$

(10 p)

Figure 3: Standard Normal distribution

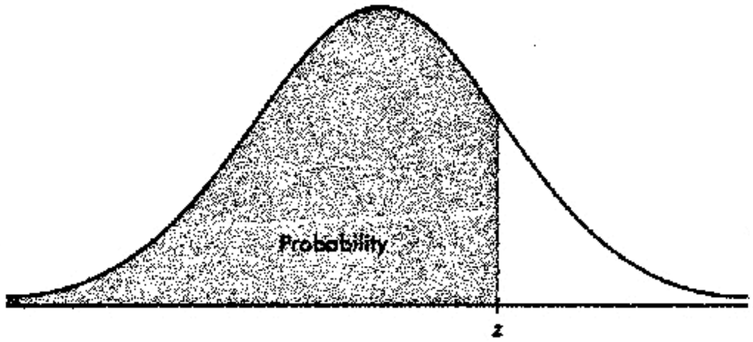


TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998