KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2018-01-15, 14:00–19:00.

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Allowed technical aids: Everything except computers and communication devices. All books, notes, old exams and similar are allowed. A calculator may be necessary.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

In Figure 1 the empirical distribution function of a sample $X_1, \ldots, X_{20}$ of 20 independent and identically distributed observations of a net worth, X, is given. Give an empirical estimate of $\mathrm{ES}_{0.15}(X)$ (you must provide a numerical answer). (10 p)

Problem 2

In a constant proportion portfolio insurance (CPPI) strategy the value of the portfolio at the beginning of the kth period is V_{k-1}, the value at the end of the kth period is V_k, and the intended guarantee at time k is G_k. Suppose that the interest rate is $r = 0$ and let R_k be the total return on a risky asset between $k-1$ and k. The return R_k is modeled as $R_k = \exp\{\mu + \sigma Z_k\}$ where $\mu = 0.05$, $\sigma = 0.2$ and Z_k has a $N(0,1)$-distribution. The strategy proceeds as follows.
At the beginning of the kth period you must decide on the leverage α to use for the kth period. Given the leverage α, the amount $\alpha(V_{k-1} - G_k)$ is invested in the risky asset (until the end of the kth period) and the remaining capital $V_{k-1} - \alpha(V_{k-1} - G_k)$ is invested in a cash account (with zero interest). Here it is assumed that $V_{k-1} > G_k$.

(a) Determine, numerically, the maximum leverage α such that $P(V_k < G_k) \leq 0.01$. (5 p)

(b) Derive an explicit formula the maximum leverage α such that

$$\mathrm{ES}_{0.01}(V_k - G_k) \leq \beta(V_{k-1} - G_k),$$

where $\beta > 0$. (5 p)

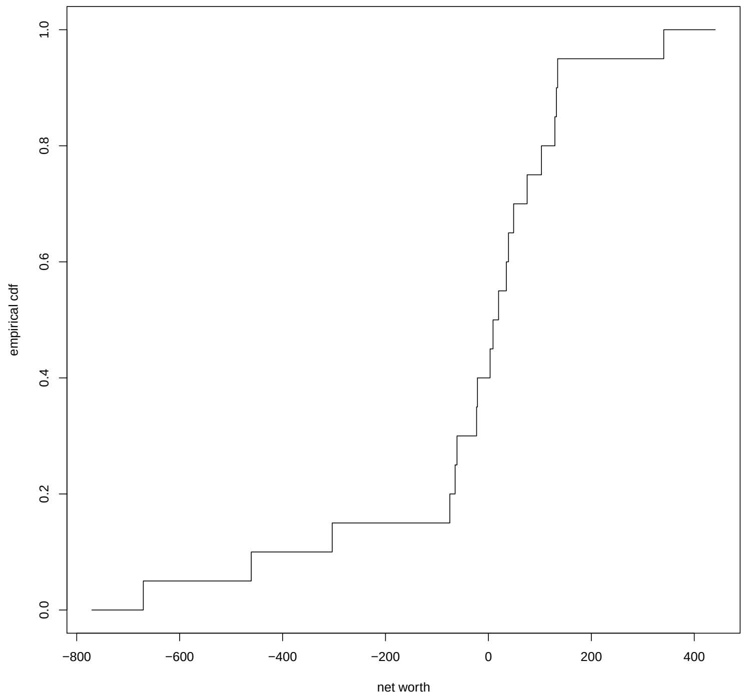

Figure 1: This figure relates to Problem 1. It gives the empirical distribution function of 20 independent observations of a net worth.

Problem 3

The risk management group at a company is developing a new scenario generator for simulating future outcomes of the logreturns (X_1, X_2) of two financial assets. Your task is to recommend one of the three model classes mentioned below. For *each* of the model classes A, B and C, you must provide arguments in favour or against using that model class for modelling (X_1, X_2) (you must argue in favour of one model class and against the other two).
In each of the model classes the model parameters will have to be estimated but you don't have to explain how to estimate the unknown parameters.

Model class A: a two-dimensional $t_\nu(\mu, \Sigma)$-distribution.

Model class B: a model consisting of a t_ν-copula and marginal distributions $N(\mu_1, \sigma_1^2)$ for X_1 and a location-scale $t_\xi(\mu_2, \sigma_2)$ for X_2.

Model class C: a model consisting of a Clayton copula and marginal distributions $N(\mu_1, \sigma_1^2)$ for X_1 and a location-scale $t_\xi(\mu_2, \sigma_2)$ for X_2.

To argue in favour or against each of the three model classes you have access to the following information.

- Quantile-quantile plot (Figure 2, left) of the empirical distribution of a sample from X_1 with respect to a standard normal reference distribution.

- Quantile-quantile plot (Figure 2, right) of the empirical distribution of a sample from X_2 with respect to a standard normal reference distribution.
- Estimate of Kendall's tau, $\tau(X_1, X_2) \approx 0.33$.
- Esimate of upper tail dependence, $\lambda_U(X_1, X_2) \approx 0.32$,
- Estimate of lower tail dependence $\lambda_L(X_1, X_2) \approx 0.31$.

The estimated values can be assumed to be estimated with an error of at most ± 0.02. (10 p)

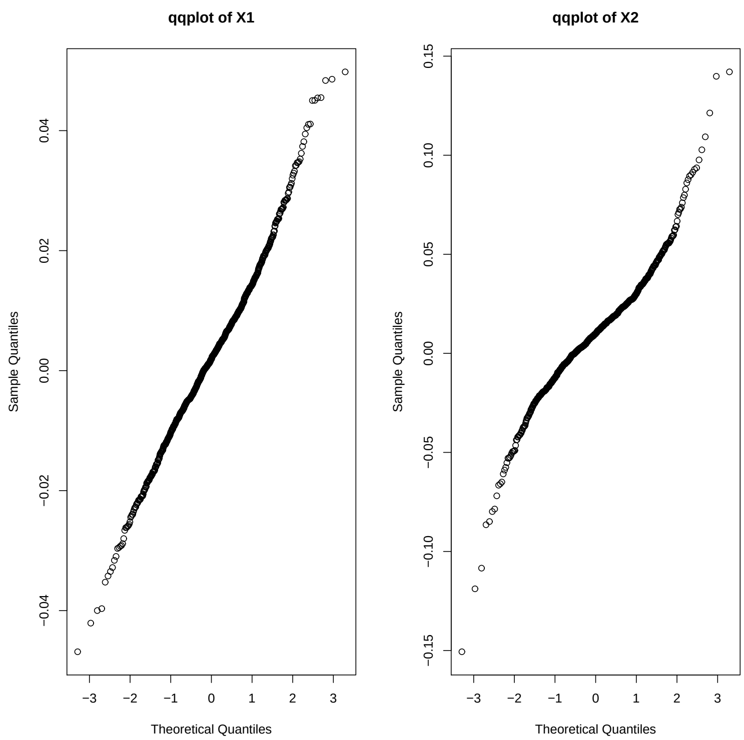

Figure 2: This figure relates to Problem 3. *Left:* quantile-quantile plot of the empirical distribution of a sample from X_1 with respect to a standard normal reference distribution. *Lower:* quantile-quantile plot of the empirical distribution of a sample from X_2 with respect to a standard normal reference distribution.

Problem 4

A company is involved in n different projects and X_i denotes the net-worth (one year from now) of the ith project, $i = 1, \dots, n$. Each project has to contribute to the company's risk buffer with amount corresponding to $\mathrm{VaR}_p(X_i)$, where the level p is to be determined. Consequently, the company's risk buffer will be

$$\mathrm{VaR}_p(X_1) + \cdots + \mathrm{VaR}_p(X_n).$$

The management of the company has decided that the company's risk buffer must be equal to $\mathrm{VaR}_{0.01}(X_1 + \cdots + X_n)$ (the 1%-Value-at-Risk of the company's total net worth).

(a) Determine a formula for p under the assumption that $(X_1, \dots, X_n)^T$ follows an elliptical distribution with location parameter $(0, \dots, 0)^T$ and dispersion matrix Σ. (Your answer may include the cdf and quantile function of a symmetric random variable Y_1.) (6 p)

(b) Determine a numerical value for p under the assumption that $X_1, \dots, X_n$ are independent $N(0, 1)$ random variables and $n = 60$. (A table of the standard normal cdf is given at the end of the exam). (4 p)

Problem 5

Let $X_1, X_2, \dots$ be independent and identically distributed random variables, with cumulative distribution function $F(x) = 1 - (1 + x)^{-3}$, $x > 0$, representing daily claim amounts of insurance claims. The insurer may be interested in purchasing a re-insurance contract that is activated if the maximum weekly (7 day) claim amount over one year (52 periods of 7 days each) exceeds a high level. More precisely, the re-insurance contract is activated if

$$\max(X_1 + \dots + X_7, X_8 + \dots + X_{14}, \dots, X_{358} + \dots + X_{364}) > 100$$

Determine the probability that the re-insurance contract is activated. Any assumptions and approximations used must be motivated. (10 p)

Figure 3: Standard Normal distribution

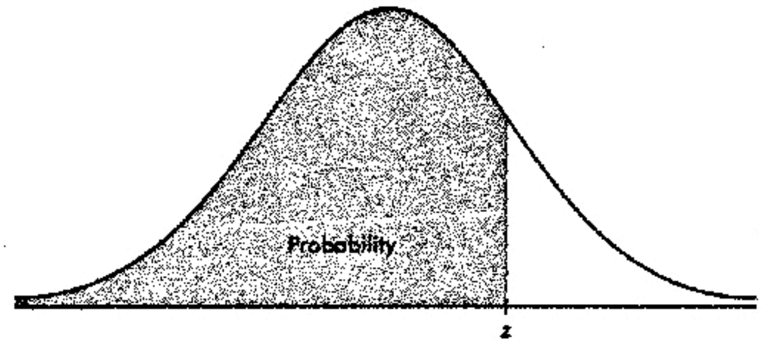

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998