KTH Matematik

EXAMINATION IN SF2980 RISK MANAGEMENT, 2018-04-03, 8:00–13:00.

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Allowed technical aids: Everything except computers and communication devices. All books, notes, old exams and similar are allowed. A calculator may be necessary.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

An investor invests SEK100 in a financial asset. Let Z_1 denoted the logreturn of the asset from today until tomorrow and $Z_{-99}, \dots, Z_0$ the observed historical logreturns of the financial asset, for the previous 100 days. Assume that all the logreturns are independent and identically distributed. A qq-plot of the historical logreturns $Z_{-99}, \dots, Z_0$ against a standard normal reference distribution is given in Figure 1. Compute an empirical estimate of $\mathrm{ES}_{0.05}(X)$ where X represents investors the net worth tomorrow. You may assume that the interest rate from today until tomorrow equals zero.

(10 p)

Problem 2

A company with two lines of business has two portfolios, one for each business line, both of which are exposed to the same set of risk factors. Suppose that the vector of risk factor changes $\mathbf{Z} = (Z_1, \dots, Z_d)$ follows a centered elliptical distribution (i.e. the location parameter is zero) and that future net worth of the two portfolios can be written as

$$X_1 = \mathbf{w}_1^T \mathbf{Z}, \quad X_2 = \mathbf{w}_2^T \mathbf{Z},$$

where $\mathbf{w}_1, \mathbf{w}_2$ are the monetary portfolio weights of the two porfolios, respectively. The risk management team has already computed the following quantities:

$$\mathrm{VaR}_{0.05}(X_1) = 23.5, \quad \mathrm{VaR}_{0.01}(X_1) = 45.4, \quad \mathrm{VaR}_{0.05}(X_2) = 110.6$$

Compute $\mathrm{VaR}_{0.01}(X_2)$. (10 p)

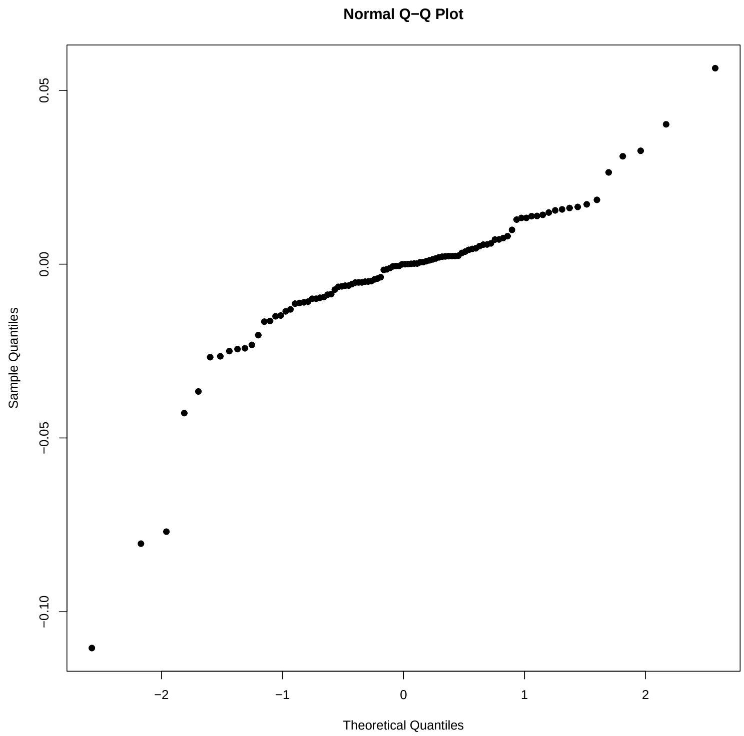

Figure 1: This figure relates to Problem 1. It gives the qq-plot of the historlcal logreturns (y-axis) against the standard normal quantiles (x-axis).

Problem 3

The risk management group at a company is developing a new scenario generator for simulating future outcomes of the vector of logreturns (X_1, X_2, X_3) of three financial assets. Your task is to recommend one of the three dependence models mentioned below, based on the available information. For *each* of the models A, B and C, you must provide arguments in favour or against using that model for modelling the dependence of (X_1, X_2, X_3); you must argue in favour of one model and against the other two. The available information on which the selection must be based is

Kendall's tau:

$$\tau(X_1, X_2) = 0.2, \quad \tau(X_1, X_3) = 0, \quad \tau(X_2, X_3) = 0.$$

Upper tail dependence:

$$\lambda_U(X_1, X_2) = 0, \quad \lambda_U(X_1, X_3) = 0, \quad \lambda_U(X_2, X_3) = 0.$$

Lower tail dependence

$$\lambda_L(X_1, X_2) = 0.25, \quad \lambda_L(X_1, X_3) = 0, \quad \lambda_L(X_2, X_3) = 0.$$

The three models are given as follows:

Model A: (X_1, X_2) has a two-dimensional Clayton copula with parameter $\theta = 0.5$ and (X_1, X_2) is independent of X_3.

Model B: (X_1, X_2, X_3) has a three-dimensional Clayton copula with parameter $\theta = 0.5$.

Model C: (X_1, X_2, X_3) has a t-copula with degrees of freedom parameter $\nu = 2.5$ and linear correlation parameters $\rho_{12} = 0.31$, $\rho_{13} = 0$, $\rho_{23} = 0$.

(10 p)

Problem 4

Let $X_1, X_2, \dots$ be a sequence of independent and identically distributed insurance claims with cumulative distribution function $F(x) = 1 - (1 + x)^{-4}$, $x > 0$. The weekly aggregated claims are given by

$$\begin{aligned} W_1 &= X_1 + \cdots + X_7, \\ W_2 &= X_8 + \cdots + X_{14}, \\ &\vdots \\ W_{52} &= X_{358} + \cdots + X_{364}. \end{aligned}$$

A re-insurance contract pays, after one year, the amount

$$S = \sum_{i=1}^{52} \max(W_i - 10, 0).$$

Compute (at least approximately) the fair price, $E[S]$, of the re-insurance contract. Any assumptions and approximations used must be well motivated. (10 p)

Problem 5

Consider a homogeneous portfolio of n loans, where the loss-given-default for each loan is $\mathrm{LGD}_i = 1$ for each $i = 1, \dots, n$. Let the default indicators $X_1, \dots, X_n$ be distributed according to a Beta mixture model, where the mixing variable Z follows a Beta(α, β) distribution. Determine the conditional expectation of the total number of defaults given that, among the first k loans, we observe m defaults. That is, determine

$$E\left[\sum_{i=1}^{n} X_i \mid \sum_{i=1}^{k} X_i = m\right],$$

where $0 \leq m \leq k \leq n$.
HINT: Compute first $E[Z \mid \sum_{i=1}^{k} X_i = m]$. (10 p)

Figure 2: Standard Normal distribution

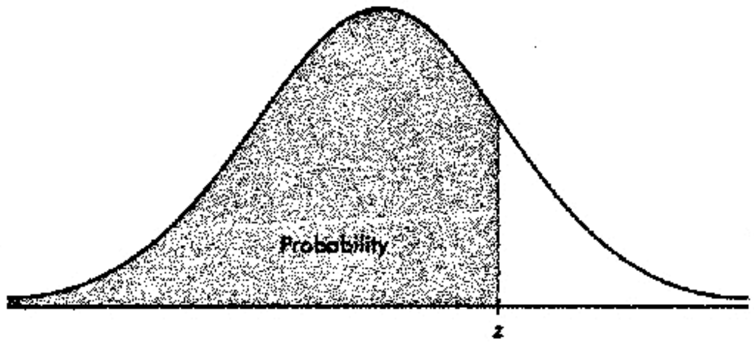

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998