EXAMINATION IN SF2943 TIME SERIES ANALYSIS

Date: 2016-08-19, 08:00-13:00

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Allowed technical aids: Calculator and "Formulas and survey, Time series analysis" by Jan Grandell, without notes

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

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Problem 1

The sample PACF for the first 10 lags from a data serie of 1 000 observations from a causal AR(p) process is given by

Lag	PACF
1	0.5587
2	-0.5342
3	-0.0427
4	0.0169
5	0.0178
6	0.0027
7	-0.0498
8	-0.0084
9	-0.0422
10	0.0075

(a) Argue that it is reasonable to assume that the order $p = 2$. (3 p)

The 15 first observations in the sample are

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-0.2753, 0.1143, 0.1430, 0.4547, -0.0498, -0.8943, -1.3266,
-0.3958, 0.2673, 0.3241, 0.7604, 0.5766, 0.1696, 0.5574, 0.0014
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(b) Use these 15 observations to find the Yule-Walker estimates of the parameters in a casual AR(2)-model. (7 p)

Problem 2

The causal AR(1) process defined by

$$X_t - \varphi X_{t-1} = Z_t,\ \{Z_t\} \sim \mathrm{WN}(0, \sigma^2),$$

can be written

$$X_t = \sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}$$

for a sequence $\{\psi_j\}$ satisfying

$$\sum_{j=-\infty}^{\infty} |\psi_j| < \infty.$$

(a) Determine the sequence $\{\psi_j\}$. (5 p)

(b) Show that the ACVF for the causal AR(1) process above is given by

$$\gamma_X(h) = \sigma^2 \frac{\varphi^{|h|}}{1-\varphi^2},\ h = 0, \pm 1, \pm 2, \ldots$$

(5 p)

Problem 3

Let $\{X_t\}$ be the times series defined by

$$X_t + 0.60X_{t-1} - 0.16X_{t-2} = Z_t + 0.15Z_{t-1},\ \{Z_t\} \sim \mathrm{WN}(0, 0.4).$$

(a) Show that $\{X_t\}$ is a well defined ARMA$(2,1)$ process. (4 p)

(b) Is $\{X_t\}$ a causal process? (3 p)

(c) Is $\{X_t\}$ an invertible process? (3 p)

Problem 4

Consider the MA(1) process

$$X_t = Z_t + 0.2Z_{t-1},\ \{Z_t\} \sim \mathrm{WN}(0, 0.1).$$

The first 3 observations X_1, X_2 and X_3 from a sample generated by this MA-model are

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0.6301, 0.2652, 0.7592
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Determine the one-step predictor $\hat{X}_4$. (10 p)

Problem 5

Let $\{\mathbf{X}_t\}$ be a 2-variate AR(1) process:

$$\mathbf{X}_t - \Phi_1 \mathbf{X}_{t-1} = \mathbf{Z}_t, \ \{\mathbf{Z}_t\} \sim \mathrm{WN}(\mathbf{0}, \Sigma).$$

Consider this model when

$$\Phi_1 = \begin{bmatrix} \alpha & 0 \\ \beta & \beta \end{bmatrix}$$

with $\alpha, \beta \in \mathbb{R}$ and such that $\alpha, \beta \neq 0$. For which values on α and β is this a causal 2-variate AR(1) process? (10 p)