Looking Over the Hedge,
Currency Hedging of Stochastic Cash Flows

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Abstract

This thesis investigates different views on and outcomes of currency hedging of stochastic investments denoted in foreign currency. Two different kinds of investors are presented; one that wants to avoid the downside of changes in the future expected exchange rate and one that wants to avoid the downside of the entire portfolio (the investment together with the currency position). Estimates are made based on data for three different currencies and Lower partial moments is used as a risk measure. The results show that it is in most cases profitable to include options in the hedge regardless of what strategy is used. The results also show that the result due to changes in the future expected exchange rate can be affected to a much larger extent than the downside of the entire portfolio. When the models are stress tested on a situation identical to that of the recent financial crisis but with reversed correlations between assets and currencies, it is still in most cases favorable to include options in the hedge. We also see that the approach that minimises the lower partial moments of the entire portfolio is much more sensitive to changes in the correlation structure than the approach of trying to avoid results due to changes in the expected future exchange rate.

Keywords: Currency hedging, Stochastic cash flows, Lower partial moments, Currency risk

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Chapter 1

Introduction

For financial institutions, corporations and individuals, it is reasonable to consider currency hedging when awaiting future cash flows in foreign currency. A currency hedge could insure you for losses due to changes in the expected future exchange rate.

A basic example is a Europe based company that exports a product to the U.S. It receives payment in USD while the suppliers and staff are paid in EUR. This means that it will be difficult for the company to predict its revenues since it indirectly means that the USD/EUR exchange rate has to be predicted. To insure itself against the possible losses (and gains) due to movements in the exchange rate, the company can buy forward contracts which give the right, and obligation, to exchange x USD for y EUR at a specific time in the future. This way the company will know today what price to set in USD for future sales in order to make a profit.

Unfortunately there are many occasions when the size of the future cash flow is unknown. Consider that we believe that we will sell products to the value of z USD at time t. We therefore enter a forward contract to exchange z USD for SEK at time t. If we then sell products to a value less then z we will have to buy USD to be able to meet our forward contract. If the exchange rate at time t differs from the forward rate (for time t quoted at time 0), which it is likely to do, two things can happen. Either we make a profit on the currency position, if the USD is now stronger, or we make a loss on the currency position, if the USD is weaker. The opposite will be true if we sell products to a value greater then z (stronger dollar equals loss and v.v.).

An alternative to forwards is options. An option gives you the right, but not the obligation, to buy (call option) or sell (put option) a certain amount of e.g. euro for w USD/EUR (w is called the strike price of the option) at a specific date in the future. Of course, you will only use your right to buy USD at the given rate if, at the end of the contract, this exchange rate is better than the one you can get in the market (the spot rate). By using options you can choose to hedge a larger amount than z without the risk of having to buy USD to commit to the contract. The downside is that options can be expensive.

The discussion above also applies to investments in equity and other financial instruments. The future value of the shares is unknown for the investor at the time of purchase.

Imagine that you have invested in for example a U.S. government bond. The bond itself comes with no (or extremely low) risk. This means that all your risk will come from the currency exposure, and currency is not what you chose to invest in to begin with. In this case the currency exposure should of course be hedged. Also, you will know for sure what your future cash flows will be. Therefore the amount you should hedge is clear and there is no need to pay for options since forwards will be cheaper.

If you on the other hand have invested in a highly volatile stock, the answer is not as clear. For one, it is very difficult to predict the future stock price, hence also how large to make the hedge. Hedging only with options would either be very expensive or the options would have to be bought at a very high strike price. Hedging with forwards would likely, as discussed before, lead to having to buy or sell USD at the expiry date due to over/under-hedging. The worst case scenario when hedging with forwards is that the underlying asset crashes at the same time as the euro becomes very expensive. The investor then both experiences a large loss in the stock and has to pay for the result of the over-hedge. This does not necessarily mean that the exposure should not be hedged, but that the size and composition of the hedge should be chosen taking the mentioned factors into account.

In this thesis we discuss and analyse the effects of different hedging strategies. This requires first considering the purpose of the hedge. One purpose could be to decrease the risk of the entire portfolio (based on historical performance), taking advantage of the correlations between the currency and asset (traditional portfolio optimisation). This implies taking a speculative position in foreign currency, meaning one must have an opinion about the development of the exchange rate. Another view is that the investor is already satisfied with the distribution of the chosen asset but wants to eliminate the currency risk. We also analyse whether it is worth including options in the hedge, different combinations of forwards and options and different hedge levels.

Since needs and preferences varies between investors, the goal of this thesis is not to find the ultimate formula but to provide insight in the issue and present some clear examples of how different hedging strategies (and hedging purposes) can result in different outcomes under different scenarios.

Chapter 2

Research

The concept of hedging, and in particular hedging currency risk, is broadly examined in the literature. However, in the pre-studies to this thesis, no studies have been found that focus solely on hedging the risk that originates from the indirect currency position of a foreign investment. Instead, previous empirical and theoretical studies focus on (roughly speaking), minimising the variance, maximising the profit or minimising the downside risk by finding the optimal weights of hedged and unhegded positions of a portfolio. Chen et al. [2001] summarise this with that the studies take either a risk-minimising approach or a utility-maximising approach. Examples of risk-minimising are the minimum variance hedge found in Kerkvliet and Moffet [1991] and the minimum lower partial moments hedge used by Lien and Kuen Tse [2001]. Utility-maximization can be found in Howard and D'Antonio [1984], where the hedge level is chosen so as to maximise the sharpe ratio.

Kerkvliet and Moffet [1991] are the first to derive an expression for minimum variance hedging of a stochastic cash flows using future contracts. They construct a portfolio consisting of a hedged and an unhedged position and derive the optimal hedge ratio that minimises the portfolio variance. When it comes to minimising the volatility of foreign stock portfolios by including currency hedging, Campbell et al. [2010] conduct a large comprehensive study. They study the currency exposure that minimises the volatility for seven different markets and the corresponding currencies. Their results may be summarised as that the Australian dollar, Canadian dollar, Japanese Yen and British pound are positively correlated with the equity markets and thus an investor should be negatively exposed to these currencies. However, the euro, the Swiss franc and US dollar are negatively correlated with the world's stock markets and an investor should thus be positively exposed to these currencies. They draw the conclusion that a US investor should be at least fully hedged, and perhaps over-hedged, to all foreign currency exposures, with the exception of the euro and Swiss franc, which shall be partially hedged. However, some criticism may be raised against using variance as the risk measure. Harris and Shen [2004] show that minimum variance hedging of currency portfolios tends to increase the portfolio kurtosis and thus the effects on more general risk measures such as VaR and CVaR are uncertain. For non-financial firms variance may not be the appropriate risk measure since corporate managers seems to be more concerned about variability in losses than in gains [Adams and Montesi, 1995]. Analogously with a more qualitative approach Stulz [1996] argues that the goal of risk management should be to eliminate costly lower tail outcomes rather than minimising the volatility.

Even though the concept of currency hedging is broadly examined in the literature most previous studies use only forwards (or futures) as hedging instruments and the literature that covers the choice between options and forwards for currency hedging is limited. Lien and Kuen Tse [2001] do attack the problem. They examine the instruments' effectiveness using lower partial moments to measure the downside risk of a portfolio. The approach is intuitively attractive since an investor probably would be most concerned about downside risk. It is shown that currency forwards almost always have a better hedging effectiveness than currency options. According to Lien and Kuen Tse [2001], options only outperform forwards for very optimistic investors (i.e. those who have high target returns), who are not more concerned about large losses than small ones. However, one criticism that can be raised against the results of Lien and Kuen Tse [2001] is that they only study an investment period of five days. Assuming that an investor has a longer investment horizon the short hedging period would lead to new hedging positions each week, both affecting the result from the currency exposure and generating transaction costs. It is a fair assumption that options could be more attractive for longer hedging horizons since assets are more volatile for longer periods.

A similar approach is taken in Albuquerque [2007] which applies different models to measure downside risk for different characteristics of the hedger (one that faces bankruptcy costs, one with loss aversion and one who gets tax benefits for hedging). This study incorporates transaction costs, and favours options (it uses the same transaction cost for both options and forwards). Albuquerque [2007] shows that forwards are more effective as hedging instruments, even though options are favoured in the models.

Chapter 3

Theory

3.1 Forwards

As mentioned in the introduction, entering a currency forward contract means taking on the obligation to exchange one currency for another in the future at a given exchange rate. Imagine entering a forward contract to sell foreign currency f and buy domestic currency d a year from now at the known price F denoted in domestic currency. The cost of this transaction should be the same as selling f now, only with the difference in interest rate between the two countries. If the one year interest rate is higher in the home country and the exchange rate will not change, then an investor would prefer to hold domestic currency to foreign. For the market to be arbitrage free the future currency exchange rate must be different from today's, meaning that you will have to pay more d/f in the future. Let X be the exchange rate today (denoted domestic/foreign), then the formula used to calculate the forward prices is

$$F = Xe^{r_f - r_d} \tag{3.1}$$

where r_f and r_d are the one year continuous interest rates.

3.2 Options

The most famous formula used to price options is the Black-Scholes formula. It is based on the assumption that the spot price at expiry is (log-) normally distributed around the forward price, with a given standard deviation (volatility). From that distribution the expected pay-off for different strike prices can be calculated. These expected pay-offs are then discounted to give the price of the option. Unfortunately, stock prices and currencies are rarely (log-) normally distributed, but often better described by for example a t-distribution. Despite of this fact and due to some nice properties of the normal distribution, the Black-Scholes formula is still widely used, but the market prices differ from those given by the formula. A widely used concept is to speak of option prices in implied volatilities. This is based on the action of taking the (market) prices of options and from those prices calculate the volatilities using the Black-Scholes formula.

Let r_f (r_d) be the foreign (domestic) risk free interest, X_0 today's exchange rate with volatility σ and $N(.)$ the cumulative standard normal distribution. Then a European currency put option with strike price K and time T to maturity will according to the Black-Scholes formula have the price p equal to

$$p = Ke^{-r_d T} N(-d_2) - X_0 e^{-r_f T} N(-d_1) \tag{3.2}$$

where

$$d_1 = \frac{\ln(S_0/K) + (r_d - r_f - \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sqrt{T}.$$

For a complete derivation of the Black-Scholes formula for currency options we refer to Björk [2004]. Historical data for the currency markets is often saved for at-the-money options and other fixed strike levels. From this data the actual (market) option prices can be calculated.

3.3 Lower Partial Moments

Lower partial moments (LPM) were introduced in its current form by Bawa (1975), though a version of LPM called semi-variance had its supporters as a risk measure earlier on. Even Markowitz, the founder of modern portfolio theory and mean-variance theory, argued in Markowitz [1959] that semi-variance (or LPM) was superior to variance as a risk measure due to its focus on downside risk. Semi-variance may be seen as "the variance below some target return", i.e. the second moment below some target return. LPM can be seen as a generalisation since it covers any moment of choice. It is defined as

$$L_{c,n} = \int_{-\infty}^{c} (c - w)^n dF(w) \tag{3.3}$$

where *F(w)* is the distribution function for W and c is some target return. Setting n=2 gives the expression for semi-variance. LPM has the attractive characteristic that it captures the preferences of an investor who wants to minimise the risk of getting below some target return. The exponent n determines how risk averse the investor is; $n = 1$ implies risk neutrality and the measure will give the average deviation from c. Larger values of n correspond to a risk averse investor. In this thesis when we speak of LPM we mean that $n = 2$ and $c = 0$, hence the LPM is the sum of the squared negative deviations from 0.

3.4 Empirical Estimate of the LPM

The expression just presented contains the true distribution function. This is often unknown and for practical implementation expressions for the empirical distribution function and for the empirical estimate of the LPM are needed. Let W be a stochastic variable with true distribution function $F(W)$ and $w_1, ..., w_m$ be observations of $W_1, ..., W_m$. Then the empirical distribution function is defined as

$$\tilde{F}(W) = \frac{1}{m} \sum_{i=1}^{m} I_{W_i \leq w} \tag{3.4}$$

where

$$I_{W_i \leq w} = \begin{cases} 1 & \text{if } W_i \leq w \\ 0 & \text{if } W_i > w. \end{cases}$$

Now the empirical estimate of the LPM can be calculated as

$$\tilde{L}_{c,n} = \frac{1}{m} \sum_{i=1}^{m} (c - w)^n I_{w_i \leq c} \tag{3.5}$$

where

$$I_{w_i \leq c} = \begin{cases} 1 & \text{if } w_i \leq c \\ 0 & \text{if } w_i > c. \end{cases}$$

3.5 Conditional Value-at-Risk

Conditional Value-at-Risk is a quantile risk measure. It measures the average loss for the worst α percents of the historical results. First we must define Value-at-Risk (VaR). Let $\alpha \in (0, 1)$ be some confidence level of the cumulative distribution function F for the stochastic variable X, then

$$\mathrm{VaR}_\alpha = \sup\{x \in R : F_X(x) \leq \alpha\}. \tag{3.6}$$

Now CVaR_α is the average size of the outcomes lower than VaR_α.

$$\mathrm{CVaR}_\alpha = \mathrm{E}[X | X \leq \mathrm{VaR}_\alpha] \tag{3.7}$$

Chapter 4

Models

In this section four different approaches to currency hedging are presented and expressions for the corresponding LPM's are derived. The approaches are chosen in order to represent both different investment strategies and different views on the goal with the hedge. One view is that the entire risk taking should be focused on the underlying asset and that the risk from the foreign currency exposure should be minimised (approach 1). Another view is that the hedge levels should be set so as to minimise the downside risk of the entire portfolio (approach 2). Approach 1 and 2 are also modified to sell off 50% of the forward position when the implied volatility for the exchange rate exceeds some threshold value (approach 3 and 4). After reviewing the behaviour of the data and results of the optimisations (see further down) a general approach (5) has been chosen using 80% forwards and 20% options (and rehedging as we fall off the goal with $\pm 10\%$). This approach is tested to see how a "fixed" hedging strategy compares to the optimised hedge levels. Along with the models presented below we will also use an approach with no hedge as benchmark. For approach 1, approach 2 and the no hedge approach we also calculate the portfolios' CVaR's on the 95%, 99% and 99.9% levels and the correlations between the asset and the currency for the 5% worth outcomes. This is to see how different hedging strategies affect the tails of the investments' distributions. However, we choose not to do this for approach 3, 4 and 5 since the result would be intangible. We also perform a stress test on the models where we take the estimated optimal hedge levels and see how those perform under a scenario identical to that of the recent financial crisis but with reversed correlations between the assets and currencies.

4.1 The investment scenario

Now we will define the investment scenario that is used for the different hedging approaches. Let Y_t be the value of an asset at time t denoted in a foreign currency. An investment is made in the asset at $t = 0$ and held until $t = T$. The period studied consists of 250 trading days (one year) and hence $T = 250$. Since the future value of the investment is unknown, Y_t is stochastic for $t = 1, ..., t = T$. The value of Y_0 is of course known today. The exchange rate at any time t is X_t, denoted domestic/foreign. X_t is of course also stochastic for $t = 1, ..., t = T$. If no hedge is made the amount $Y_0 X_0$ is invested in domestic currency today and the value of this investment in domestic currency at time T is $Y_T X_T$.

Let us define β_f as the hedge ratio for forwards and β_o as the hedge ratio for options, both expressed as a fractions of Y_0. The above definitions means that if $\beta_f > 0$ we take a position in forward contracts to sell foreign currency and buy domestic (or correspondly for options, if $\beta_o > 0$ we buy options allowing us to exchange foreign currency for domestic).

4.2 Approach 1: Minimising the downside of the currency exposure

It is now time to introduce the measure that has been chosen to define losses due to imperfect currency hedges in this thesis. The optimal scenario for an investor who wants no currency risk is to have a fixed rate at to which the whole investment value can be exchanged (here from foreign to domestic currency) at T. Since today, the market expects the future exchange rate at T to be the forward rate F_0^T, this is the reasonable exchange rate to set as an optimum. However, since the amount to be exchanged Y_T is stochastic, we are unlikely to set our hedge perfectly. If Y_T exceeds the hedge level $Y_0\beta_f$, we will have to exchange that amount at the spot exchange rate, X_T (which will be favourable if $X_T > F_0^T$). Or, if Y_T is lower than $Y_0\beta_f$, we will have to buy that amount of foreign currency at the spot rate X_T to meet our forward agreement (which will be favourable if $X_T < F_0^T$). Hence the deviation Φ from the perfect hedge is

$$\Phi_{f,ABS} = (Y_T - Y_0\beta_f)(X_T - F_0^T)$$

or as a fraction of $Y_T F_0^T$

$$\Phi_f = \frac{(Y_T - Y_0\beta_f)(X_T - F_0^T)}{Y_T F_0^T}.$$

Negative results can occur both when we are under-hedged

$$\frac{Y_T}{Y_0} - \beta_f > 0 \text{ and } X_T - F_0^T < 0$$

and when we are over-hedged

$$\frac{Y_T}{Y_0} - \beta_f < 0 \text{ and } X_T - F_0^T > 0.$$

Hence, we have chosen to define losses as negative results; $\Phi < 0$. The formula for the result can be re-written as

$$\Phi_f = \frac{Y_T X_T + Y_0\beta_f(F_0^T - X_T) - Y_T F_0^T}{Y_T F_0^T}$$

where the value of an unhedged portfolio at time T is $r_T = Y_T X_T$, the result of the forward position is $r_F = Y_0\beta_f(F_0^T - X_T)$ and the value of a portfolio if the investment is hedged perfectly is $r_{opt} = Y_T F_0^T$. Hence

$$\Phi_f = \frac{r_T + r_f - r_{opt}}{r_{opt}}.$$

Options can also be included in the hedge. Set the strike price to F_0^T, let p be the price today and T the time of maturity, then the result of the option position is $Y_0\beta_o(\max(0, X_T - F_0^T) - p)$. However, including this directly in the model would give it a small flaw, high option costs can be compensated by good option/forward performances. To avoid this problem only the positive results of the hedge are included in the result up to a hedge level of 100%, hence we define $r_o = \min(Y_0\beta_o, Y_1 - \beta_o Y_0)\max(0, X_1 - F_0^1) - p\beta_o Y_0$ and

$$\Phi_{f,o} = \frac{r_T + r_f + r_o - r_{opt}}{r_{opt}}. \tag{4.1}$$

Now since r_f is a function of β_f and r_o is a function of β_o, then $\Phi_{f,o}$ is a function of β_f and β_o; $\Phi_{f,o} = \Phi_{f,o}(\beta_f, \beta_o)$. This means that we can minimise the LPM for $\Phi_{f,o}$ over β_f and β_o. Let ϕ_1, ... ,ϕ_m be observations of Φ_1, ..., Φ_m. Then the empirical estimate of LPM_Φ (with $c = 0$ and $n = 2$) can be written as

$$\text{LPM}_{\phi_{f,o}}(\beta_f, \beta_o) = \sum_{i=1}^{m} \phi_i^2 I_{\phi_i<0} \tag{4.2}$$

where

$$I_{\phi_i\leq 0} = \begin{cases} 1 & \text{if } \phi_i \leq 0 \\ 0 & \text{if } \phi_i > 0 \end{cases}$$

and hence we have the following optimisation problem [1]

$$\begin{gathered} \min_{\beta_f,\beta_o} \text{LPM}_{\phi_{f,o}} \\ |\beta_f| + |\beta_o| \leq 2 \\ \beta_f \ , \beta_o \geq 0 \end{gathered} \tag{4.3}$$

Now let us denote the hedge levels that solve problem 4.3 as $\beta_f^{*\phi}$ and $\beta_o^{*\phi}$ and let us for notational purposes define a vector consisting of the two solutions as $\beta_{fo}^{*\phi} = [\beta_f^{*\phi}, \beta_o^{*\phi}]$.

Analogously we may derive an expression for only hedging with forwards which leads to the following optimisation problem

$$\begin{gathered} \min_{\beta_f} \text{LPM}_{\phi_f} \\ |\beta_f| \leq 2 \\ \beta_f \geq 0. \end{gathered} \tag{4.4}$$

With the same notation as above let the solution to problem 4.4 be written as $\beta_{f,f}^{*\phi}$.

4.3 Approach 2: Minimising the downside of the entire portfolio

We will now derive the target function for an investor that wants to minimise the downside risk of a portfolio consisting of a foreign investment, currency forward contracts and currency put options. The portfolio result P_f when using only forwards as hedging instruments may be written as

$$P_f = \frac{Y_T X_T + Y_0 \beta_f (F_0^T - X_T) - Y_0 X_0}{Y_0 X_0}. \tag{4.5}$$

[1] The bounds introduced for β_f and β_f have no rigorous mathematical derivation or origin but are simply based on the assumption that the investor has some maximum hedge level that he or she is not willing to exceed. When minimising Φ short selling is not allowed since this will lead to larger currency exposures.

Setting $\pi_T = Y_T X_T$, $\pi_0 = Y_0 X_0$ and $\pi_f = Y_0\beta_f(F_0^T - X_T)$ gives

$$P_f = \frac{\pi_T + \pi_f - \pi_0}{\pi_0}. \tag{4.6}$$

Options may also be included in this approach. Set the strike price to F_0^T, let p be the price today and T the time of maturity, then $\pi_o = Y_0\beta_o(\max(0, X_T - F_0^T) - p)$, $\pi_p = Y_0\beta_o p$ (the total cost of the option position) and

$$P_{f,o} = \frac{\pi_T + \pi_f + \pi_o - \pi_0}{\pi_0 + \pi_p}. \tag{4.7}$$

Now the same procedure as for approach 1 is followed. Let ρ_1, ... ,ρ_m be observations of P_1, ..., P_m, then the empirical estimate of the LPM_P can be written as

$$\text{LPM}_{\rho_{f,o}}(\beta_f, \beta_o) = \sum_{i=1}^{m} \rho_i^2 I_{\rho_i<0} \tag{4.8}$$

where

$$I_{\rho_i \le 0} = \begin{cases} 1 & \text{if } \rho_i \le 0 \\ 0 & \text{if } \rho_i > 0 \end{cases}$$

and hence we have the following optimisation problem [2]

$$\min_{\beta_f, \beta_o} \text{LPM}_{\rho_{f,o}} \tag{4.9}$$

$$|\beta_f| + |\beta_o| \le 2.$$

With the same notation used in approach 1 let the solution to problem 4.9 be $\beta_f^{*\rho}$ and $\beta_o^{*\rho}$ and let us for notational purposes define a vector consisting of the two solutions as $\beta_{fo}^{*\rho} = [\beta_f^{*\rho}, \beta_o^{*\rho}]$.

Analogously only hedging with forwards gives

$$\min_{\beta_f} \text{LPM}_{\rho_f} \tag{4.10}$$

$$|\beta_f| \le 2.$$

Also, let the value of β_f solving the optimisation problem be denoted $\beta_{f,f}^{*\rho}$.

4.4 Approach 3 and 4: Re-hedging Volatility Trigger

Sometimes an investor might want to rebalance the hedge. If the volatility increases a lot one would probably prefer using fewer forwards and perhaps more options to hedge. Let $\tau = 1, ...250$ be the days during the one year investment period. Now let us define δ as some trigger and γ as some threshold so that if $\delta > \gamma$ the investor will rebalance the hedge by closing s percent of the forward position. Here, only one re-hedge for each period will be considered; as soon as $\delta > \gamma$ the investor will close/sell s percent of the forward contracts and then not change the hedge regardless of what happens with δ.

[2] Since we want to minimise the downside of the portfolio in this approach this means that we sometimes wants to increase our exposure to the foreign currency and hence short selling is allowed

Let

$$s(\tau) = \begin{cases} s & \text{if } \delta_\tau > \gamma,\ \delta_{\tau-1} < \gamma, ..., \delta_0 < \gamma \\ 0 & \text{else.} \end{cases}$$

Now, the result of the forward position can be written as

$$(1 - s(\tau))\beta_f(X_1 - F_o^1) + s(\tau)\beta_f(X_\tau - F_0^1) =$$

$$= \beta_f(X_1 - F_0^1) + s(\tau)\beta_f(X_\tau - X_1)$$

where $\beta_f(X_1 - F_0^1)$ is the result of the original forward position. The result generated from the sell back can then be defined as $r_{sb} = s(\tau)\beta_f(X_\tau - X_1)$.[3]

Approach 1 and 2 above are optimised both with and without the just explained re-hedging method, with implied volatility as the trigger and the trigger levels USDSEK 0.14, EURSEK 0.075 and EURUSD 0.14. Including the result of the sell back in Φ and P gives:

$$\Phi_{f,o,sb} = \frac{r_T + r_f + r_{o+} + r_{sb} - r_{opt}}{r_{opt}} \tag{4.11}$$

$$P_{f,o,sb} = \frac{\pi_T + \pi_f + \pi_0 + \pi_{sb} - \pi_o}{\pi_0 + \pi_p} \tag{4.12}$$

This leads to similar optimisation problems as (4.3), (4.4),(4.9) and (4.10). In this thesis s has been chosen to be 50%.

4.5 Approach 5: Fixed Hedge-Ratio, Hedge-Level Trigger

As soon as the underlying asset changes value, the hedge level changes as well. Therefore if the asset is volatile it is natural to consider re-setting the hedge to a given level a couple of times during the investment period. One way of doing so is to set the hedge to a specific level β_f^{fix} and then reset it to β_f^{fix} as soon as it deviates more than a certain relative threshold level ϵ from β_f^{fix}. Let k_0, k_1,..., k_n be the times when the forward position is changed such that for any t_j where $k_i < t_j < k_{i+1}$, the following must hold $|\beta_f^{fix} Y_{t_j} - \beta_f^{fix} Y_{k_i}| < \epsilon Y_{t_j}$ and $|\beta_f^{fix} Y_{k_i} - \beta_f^{fix} Y_{k_i+1}| > \epsilon Y_{t_j}$. Also let h_0, h_1,..., h_n be the sizes of the changes in forward positions at the times k_0, k_1,..., k_n. Then $h_0 = \beta_f^{fix} Y_{k_0}$, $h_1 = \beta_f^{fix} Y_{k_1} - h_0$, $h_2 = \beta_f^{fix} Y_{k_2} - h_1$ and so on. The total results of the forward positions can now be written as

$$r_{fix} = \sum_{t=k_0}^{k_n} h_t(F_t^{t+250} - X_T). \tag{4.13}$$

Hence

$$\Phi_{fix} = \frac{r_T + r_{fix} + r_{o+} - r_{opt}}{r_{opt}} \tag{4.14}$$

and

$$P_{fix} = \frac{\pi_T + \pi_{fix} + \pi_0 - \pi_o}{\pi_0 + \pi_p}. \tag{4.15}$$

In this thesis we will as previously mentioned set $\beta_f^{fix} = 0.8$ and $\epsilon = 0.1$.

[3] It is worth mentioning that forward contracts are not closable before expiry but that the same result can be achieved by buying foreign currency at τ.

Chapter 5

Data

The data is provided by Nordea e-markets and Bloomberg. The time interval ranges from 1995 to 2010 but differs slightly for different data series. This is due to the fact that some quotes during the period are missing for some assets and that these dates are simply removed from the data sets. The data consists of prices of underlying assets (stock indices and one bond index), foreign exchange rates (between EUR, SEK and USD) and one year implied volatilities and interest rates (LIBOR and STIBOR) for these exchange rates. For the EUR, GBP and USD the STIBOR rates are used. The Swedish LIBOR cannot be used due to lack of data so the STIBOR is used instead. One might argue about which interest rate should be used as the "risk-free rate" for pricing options and forwards. It may be natural to consider the rates from government bonds to be the correct benchmark. Here we rely on Hull [2005] pp. 77 that argues that financial institutions generally use the LIBOR-rates when pricing derivatives and that the investor is likely to buy the hedging product from such an institution.

In this thesis we will as previously mentioned study an investment period of one year. Since we have data for approximately 16 years this means that we could only get 16 disjoint one year periods. Of course would that be too few to draw any conclusions. In order to get a sufficient amount of data points we build data sets of rolling one year periods (i.e. 1999-01-01 to 2000-01-01, 1999-01-02 to 2000-01-02 and so on).

The data includes two highly volatile periods; the IT-Boom ranging from approximately August 1999 to August 2002 and the recent financial crisis. One might argue that events such as the fall of Lehman Brothers or September 11^{th} 2001 should be labelled outliers and be removed from the time series. In this thesis we have decided to include such events. It would not make sense to remove outliers when one area of interest of this study is the behaviour of the tails of the distributions.

To be able to stress test the model we have built one additional data set consisting of data from the financial crisis but with reversed correlations between the currencies and assets. To do this we took the last 600 data points for each investment scenario and changed places between the foreign and domestic currencies. Take for example S&P 500 and a SEK investor; to be able to reverse the correlations we handled the data as if S&P 500 was denoted in SEK instead of USD and that we had a USD investor instead of a SEK investor.

5.1 Option Pricing

To calculate the options prices for this thesis it would be preferable to use the volatility for options with strike at the forward price. Though in absence of such specific data the at-the-money volatilities[1], which differ very little from the forward volatilities, are used.

[1]Strike price equal to spot price day 0, see chapter 3 Theory for more information on implied volatility.

The implied volatilities as well as the interest rates are used to calculate the prices of options with one year duration. These options can perhaps not be found on the exchanges (since standardised one year currency options are not emitted each day) but can be seen as options from the OTC market where large investors can trade this kind of contracts and therefore a realistic situation is still reflected.

5.2 Autocorrelation

The time series of ϕ's and ρ's have very strong autocorrelations, meaning the outcomes of one result depends heavily on the historical ones. This is no surprise; the fact the results are calculated on moving data means that one outcome is based on almost the same data as the next. Autocorrelation causes some statistical difficulties since the data sample is bias depending the start date and a large amount of data is needed to find the true distribution. However, the time series describing the ϕ's will still be ergodic, due to the fact that not all data points are overlapping. This means that the estimates calculated in this thesis will still be consistent but not necessarily efficient. Autocorrelation only affects the variance (not the value) of a point estimate.

5.3 Data summary statistics

Table 5.1 shows a summary of statistics for the constructed data series. All values are on a yearly basis.

	Denoted in	Mean (%)	Vol (%)	Skewness	Kurtosis	Min (%)	Max (%)
OMRX Bond	SEK	6.11	3.61	-0.23	1.83	-1.42	14.25
OMS30	SEK	0.62	30.57	-0.17	1.64	-55.28	71.15
Dow Jones	USD	0.88	16.92	-0.46	2.68	-47.17	43.08
NASDAQ	USD	-3.20	28.52	-0.05	2.46	-59.53	80.40
RTS	USD	35.70	51.81	-0.27	2.84	-75.78	175.88
S&P500	USD	-2.68	19.73	-0.33	2.11	-49.20	42.85
DAX	EUR	1.59	27.99	-0.18	1.90	-57.09	73.51
IBEX	EUR	1.73	23.18	-0.30	1.81	-50.10	46.84
MI30	EUR	-3.41	23.55	-0.36	1.92	-57.34	45.99
CA40	EUR	-1.85	24.84	-0.05	2.05	-48.82	67.02
USDSEK	-	2.64	15.03	-0.28	2.28	-34.57	35.32
EURSEK	-	-2.19	4.99	-0.95	3.66	-18.66	13.51
EURUSD	-	-2.18	10.89	0.62	2.47	-20.57	26.59

Table 5.1: Summary statistics for the constructed data series, all values are calculated on a yearly basis and on moving data.

Chapter 6

Results

6.1 LPM's

In this section the LPM's of the different approaches including both forwards and options are presented. The results for using forwards only are found in Appendix B, table B.1.

EURSEK	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	395	70	0	12	1	1	75	79	0	0	84	3
OMXS30	511	5390	2	17	3867	4343	26	107	2929	3400	85	4381
DAX	88	3249	5	5	3832	3662	132	45	2251	2544	77	4005
IBEX	77	2069	5	5	2598	2467	123	67	1087	1301	59	2765
MI30	85	3259	7	6	3854	3736	205	206	1716	2148	95	4120
CA40	84	3027	6	6	3632	3480	124	68	1859	2165	80	3843
AVERAGE	207	2844	4	8	2964	2948	114	95	1640	1926	80	3186

USDSEK	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	1353	653	1	2	8	7	51	1	1	1	244	40
OMXS30	1685	6297	48	50	4634	4466	354	375	3165	3177	312	5528
Dow Jones	1631	2210	52	63	1749	1745	393	391	719	706	404	2386
NASDAQ	1677	5182	120	141	4783	4625	403	418	3254	3167	628	5917
RTS	1469	3920	334	342	4596	4524	2867	2834	1081	976	954	5478
S&P500	1666	3021	61	78	2541	2518	391	384	1293	1262	440	3356
AVERAGE	1580	3547	102	113	3052	2981	743	734	1586	1548	497	3784

EURUSD	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
Dow Jones	1268	2294	37	43	1565	1556	356	359	1082	1080	311	1983
NASDAQ	1340	5185	61	89	4216	4231	425	414	3662	3643	424	4936
RTS	1212	4151	255	259	4401	4387	2261	2258	2648	2616	697	4855
S&P500	1293	3061	44	52	2198	2194	368	371	1694	1688	339	2741
DAX	723	3883	47	43	3318	3310	204	224	2634	2645	278	4067
IBEX	743	3140	13	16	2238	2225	319	319	1758	1758	166	2814
MI30	730	3818	27	29	3246	3273	274	277	2552	2551	230	3965
CA40	730	3657	37	36	3083	3085	238	252	2396	2400	240	3799
AVERAGE	1005	3649	65	71	3033	3033	556	559	2303	2298	336	3645

Table 6.1: LPM_ϕ and LPM_ρ for the different hedging approaches when including both forwards and options. The values of the LPM's are quoted as 10^{-5}. The sub indices *SB* indicates the result of the portfolio when re-hedged according to section 4.4.

As can be seen in table 6.1 there was no improvement when using the (implied) volatility as a trigger for lowering the hedge; both LPM_ϕ and LPM_ρ remained at close to the same level in almost all cases. It is of interest to note that the LPM_ϕ for OMRX Bond Index became around twelve times larger with this strategy than with approach 1. This is of no surprise since the OMRX Bond Index return is almost deterministic and the fluctuations in the LPM_ϕ originate almost solely from the currency exposure; the position should hence be hedged to a high level to avoid a large LPM_ϕ. Further it can be noted that LPM_ϕ was much lower for the optimised portfolio when we minimised LPM_ϕ than for an unhedged portfolio. The differences in LPM_ϕ range from almost 100% for the OMRX Bond Index (both for USD and EUR investors) to 77% for a SEK investor in NASDAQ Composite Index, with an average of 94%. The reduction in LPM_ρ (when minimising LPM_ρ) ranges from 100% for a USD investor in OMRX Bond Index and 100% for a EUR investor down to 32% for a EUR investor in the S&P500 index, with an average of 49%. Analysing these results it seems as if it is much easier to reduce LPM_ϕ than LPM_ρ by optimising the hedging strategy. The fixed strategy performed rather weakly measured in LPM_ϕ for a SEK investor in EUR assets but gave even worse results than the unhedged portfolio for the MI30 Index. Further, the fixed strategy gave much better results for the more volatile currency pairs, EURUSD and USDSEK, than for the less volatile EURSEK. The strategy lowered LPM_ϕ (compared to the unhedged portfolio) with between 83% (for a USD investor in OMRX Bond Index) to 35% (for a SEK investor in the NASDAQ Composite Index), with an average of 58%.

6.2 The effects on the tail

As can be seen in table 6.1 there was an increase in LPM_ρ when minimising LPM_ϕ for some assets. This is because the currency exposure is a natural hedge to the underlying asset. For large losses the results of the forward hedge will be positively correlated, i.e. we have a negative tail correlation between the asset and the result of the forward position. Figure 6.1 below shows the portfolio results for a SEK investor in RTS. The results are sorted from the worst to the best 250-day period. The results of the forwards, options and assets are presented separately to show from where the results originate. We clearly see that there is a positive correlation between RTS and the forward position in the negative tail of the portfolio return. Similar results can be found for many of the other assets.

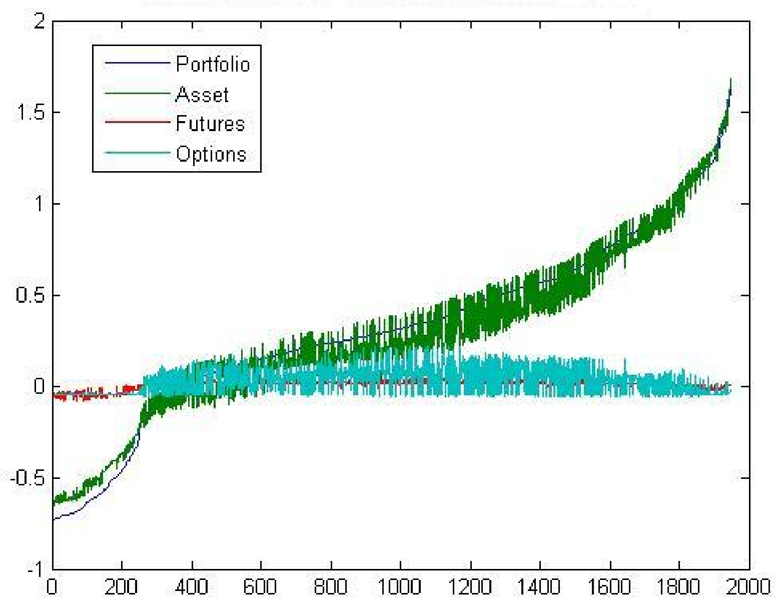

Figure 6.1: Portfolio results from minimising LPM_ϕ in RTS, sorted from worst to best and presented by component.

However, even if LPM_ρ decreased when minimising LPM_ϕ we still experienced fatter tails for some scenarios, see example in figure 6.2 that shows histograms of ϕ and ρ for S&P500. We see that the portfolio with minimised LPM_ϕ has a fatter tail than the unhedged portfolio. Since this entire analysis is based on historical data, this of course only holds true under the assumption that the way the correlations behave during tough conditions does not change. This will be covered in more detail in chapter 7.

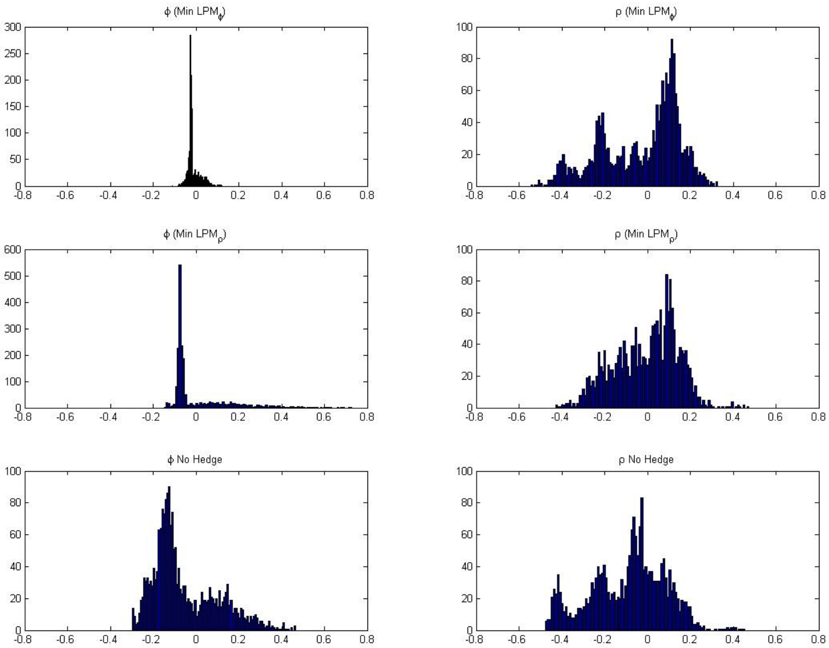

Figure 6.2: Histograms over ϕ (left) and ρ (right) for a SEK investor in S&P500 when minimising LPM_ϕ (upper), LPM_ρ(center) and no hedge (lower)

In table 6.2 the differences in $\text{CVaR}_{95\%}$ are presented, as well as the corresponding correlations for the 5% worst outcomes. We see that $\text{CVaR}_{95\%}$ decreases with between 1 and 5 percentage points when LPM_{ϕ} is minimised for some of the assets. On the other hand $\text{CVaR}_{95\%}$ improves with between 1 and 21 percentage points for some of the assets. As one might guess the differences are clearly connected to the sign of the correlation between the asset and currency for the 5% worst outcomes. For a complete list of the CVaR's see table C.1 in Appendix C.

EURSEK			USDSEK			EURUSD		
Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)
OMRX Bond	-8	3	OMRX Bond	-21	-16	Dow Jones	2	-15
OMXS30	-9	13	OMXS30	-10	4	NASDAQ	-5	0
DAX	-2	-25	Dow Jones	1	-18	RTS	1	-10
IBEX	4	-10	NASDAQ	0	0	S&P500	2	-17
MI30	5	-45	RTS	2	-7	DAX	-5	4
CA40	1	-27	S&P500	-1	-22	IBEX	-16	16
						MI30	-17	21
						CA40	-13	21

Table 6.2: The differences in $\text{CVaR}_{95\%}$ between an unhedged portfolio and a portfolio for which LPM_{ϕ} is minimised and the corresponding correlations between the assets and currencies for the 5% worst results. PP is short for percentage points.

In table 6.3 the differences in $\text{CVaR}_{95\%}$ between an unhedged portfolio and a portfolio for which LPM_{ρ} is minimised are presented. We clearly see that minimising LPM_{ρ} also has large positive effects on the negative tail of the portfolio. The improvements range between 2 percentage points for a SEK investor in DAX to 34 percentage points for a EUR investor in RTS.

EURSEK			EURUSD			USDSEK		
Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)
OMRX Bond	-8	3	OMRX Bond	-23	-16	Dow Jones	-9	-15
OMXS30	-9	13	OMXS30	-11	4	NASDAQ	-6	0
DAX	-2	-25	Dow Jones	-15	-18	RTS	-20	-10
IBEX	-5	-10	NASDAQ	-5	0	S&P500	-9	-17
MI30	-8	-45	RTS	-34	-7	DAX	-13	4
CA40	-2	-27	S&P500	-13	-22	IBEX	-20	16
						MI30	-20	21
						CA40	-18	21

Table 6.3: The differences in $\text{CVaR}_{95\%}$ between an unhedged portfolio and a portfolio for which LPM_{ρ} is minimised and the corresponding correlations between the assets and currencies for the worst 5% results. PP is short for percentage points.

6.3 The choice between options and forwards

EURSEK								
	min LPM_ϕ				min LPM_ρ			
	Forwards	Forwards & Options		Improvement	Forwards	Forwards & Options		Improvement
	$\beta_{f,f}^{*\phi}(\%)$	$\beta_f^{*\phi}(\%)$	$\beta_o^{*\phi}(\%)$	(%)	$\beta_{f,f}^{*\rho}(\%)$	$\beta_f^{*\rho}(\%)$	$\beta_o^{*\rho}(\%)$	(%)
OMRX Bond	109	107	2	4	52	31	25	-72
OMXS30	99	99	1	-2	200	200	0	0
DAX	62	61	5	1	-200	-163	-37	0
IBEX	61	59	12	3	-200	-163	-37	1
MI30	52	50	16	5	-200	-200	0	0
CA40	60	59	8	1	-200	-163	-37	0

USDSEK								
	min LPM_ϕ				min LPM_ρ			
	Forwards	Forwards & Options		Improvement	Forwards	Forwards & Options		Improvement
	$\beta_{f,f}^{*\phi}(\%)$	$\beta_f^{*\phi}(\%)$	$\beta_o^{*\phi}(\%)$	(%)	$\beta_{f,f}^{*\rho}(\%)$	$\beta_f^{*\rho}(\%)$	$\beta_o^{*\rho}(\%)$	(%)
OMRX Bond	106	102	7	42	81	52	42	-17
OMXS30	71	62	28	19	69	-11	189	37
Dow Jones	72	59	50	54	19	-27	173	56
NASDAQ	66	48	52	31	-3	-43	157	28
RTS	35	16	111	45	-129	-137	63	21
S&P500	69	56	52	51	13	-30	170	46

EURUSD								
	min LPM_ϕ				min LPM_ρ			
	Forwards	Forwards & Options		Improvement	Forwards	Forwards & Options		Improvement
	$\beta_{f,f}^{*\phi}(\%)$	$\beta_f^{*\phi}(\%)$	$\beta_o^{*\phi}(\%)$	(%)	$\beta_{f,f}^{*\rho}(\%)$	$\beta_f^{*\rho}(\%)$	$\beta_o^{*\rho}(\%)$	(%)
Dow Jones	83	68	34	29	50	-28	172	34
NASDAQ	82	72	23	11	52	-25	175	14
RTS	46	22	87	33	-128	-133	67	12
S&P500	80	66	36	26	52	-27	173	26
DAX	65	53	37	21	0	-37	163	20
IBEX	84	77	18	16	69	0	200	23
MI30	81	74	25	14	37	-12	188	21
CA40	75	66	29	14	25	-23	177	21

Table 6.4: Hedge ratios when including only forwards or both forwards and options, and the improvement in LPM when introducing options.

Table 6.4 shows the optimised hedge levles when using only forwards and when including options, as well as the improvement in LPM when including options in the hedge. The total hedge level that minimises LPM_ϕ using both forwards and options ranges from 109% for a USD investor in the OMRX Bond Index to 66% for a SEK investor in the MI30 index. The spread in hedge level between different assets is larger when only using forwards when it ranges from 109% for a EUR investor in OMRX Bond Index to only 35% for a SEK investor in the RTS Index. Generally we see that the total hedge levels when including options are higher then when only using forwards and become rather extreme for the RTS Index. The large position in options when minimising the RTS Index may be explained by the fact that it is the most volatile asset studied. Similarly the OMRX Bond Index, that is the least volatile asset, shows the smallest difference in hedge ratios. For both USDSEK and EURUSD we see large improvements in both LPM_ϕ and LPM_ρ when including options. For EURSEK, on the other hand, we see very small improvements.[1]

We can see that minimising LPM_ρ generally leads to extremely large currency exposures (ranging from +200% to -200%, which are the given limits for the optimisation) when hedging with both forwards and options. It is also of interest to notice that for a SEK investor in EUR assets the total hedge ratio is always -200%. It can also be seen that

[1] The table shows that the LPM's actually increase for some investment scenarios, though the absolute differences are under the tolerance level for the optimisation (10^{-6}) and therefore negligible.

for these investment scenarios minimising the LPM_ρ leads to short positions in options. For the currency pair USDSEK and the corresponding assets (except for the RTS Index and the OMRX Bond Index) we see that the optimal hedge tends to consist of a modest negative position in forwards (increasing the currency exposure) and a largely positive position in options (decreasing the currency exposure). With this kind of position we will make a profit when the value of the foreign currency increases a lot (so that the gain of the forwards is larger than the cost of the options) and when it decreases (since we have positive net hedge levels).

Chapter 7

Stress testing

In this chapter the results of stress testing the optimised hedge levels for the different strategies are presented.[1] The stress test was performed using data from the recent financial crisis but with reversed correlations between assets and currencies. The fixed hedge approach and the no hedge approach are also tested on this scenario.

7.1 LPM's

When using $\beta^{*\phi}_{f,o}$ during the stress test period we experienced very good results in LPM_ϕ compared to the unhedged portfolio; most of the improvements were larger than 90%. The exception was the EUR investor in OMXS30 for which we observed a much higher LPM_ϕ for the hedged portfolio than for the unhedged portfolio. The improvements were of the same magnitudes as those in-sample and hence the approach minimising LPM_ϕ seems to be stable to changes in the correlation structure. It was also noted that reversing the correlations led to very positive effects on LPM_ρ when using $\beta^{*\phi}_{f,o}$. Studying at the results for $\beta^{*\rho}_{f,o}$ we see large negative impacts on LPM_ρ for some of the assets and positive impacts for some of them. However we see that these improvements in LPM_ρ were not of the same magnitudes as the negative results. The negative results occurred for a SEK investor in EUR assets, a EUR investor in OMSX30 and for both SEK and EUR investors in RTS. In table 6.4 we see that these negative results correspond to negative net hedge ratios for $\beta^{*\rho}_{f,o}$ except for for a EUR investor in OMSX30 for which we have a positive net hedge ratio of 200%. As a result of these sometimes large negative effects on LPM_ρ the hedge levels $\beta^{*\rho}_{f,o}$ can be very sensitive to changes in the correlation structure.

The fixed strategy gave a better LPM_ϕ than the unhedged portfolio with almost the same levels of improvement as for $\beta^{*\phi}_{f,o}$. For LPM_ρ the fixed strategy improved the results for 14 portfolios and worsened it for six, compared to the unhedged portfolio. It can also be noted that the results of this strategy were much more stable to changes in the correlation structure between in-sample and out-sample for LPM_ϕ than for LPM_ρ. Lowering the hedge when the implied volatilities became high often led to larger LPM's during the stress test period, especially when minimising LPM_ϕ. Results for when only including forwards in the hedges are presented in Appendix D.

[1] The optimal hedge ratios are presented in table 6.4 (approach 1 and 2) and in table A.1 in Appendix A (approach 3 and 4).

EURSEK												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	26	3	0	0	0	0	3	0	0	0	17	0
OMSX30	15	4356	142	21	6434	5702	2679	7122	10736	7122	36	5868
DAGS	1024	8216	10	9	5001	4910	7373	17560	17907	17560	86	5712
IBEX	1026	8391	11	15	5105	5201	7431	17703	17939	17703	101	6018
MI30	821	10808	3	3	7557	7494	10586	19440	19618	19440	52	8486
CA40	1048	9964	6	7	6382	6349	7838	19905	20186	19905	81	7265
Average	238	1662	29	9	5080	4943	5985	13622	14398	13622	22	1328

USDSEK												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	1362	770	0	1	0	0	36	0	0	0	200	17
OMSX30	1238	5512	57	133	5225	5353	340	2434	2451	2434	170	6253
Dow Jones	3525	10846	20	21	2575	2491	257	1887	1844	1887	161	5507
NASD	3503	10668	23	31	2892	2826	207	3223	2916	3223	187	5502
RTS	5067	22860	104	101	8271	8188	23892	32125	31190	32125	173	16711
S&P500	3590	11774	22	24	3080	2978	248	2478	2380	2478	155	6352
Average	1290	2755	38	52	3674	3639	4163	7024	6797	7024	74	1784

EURUSD												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
Dow Jones	1328	7819	4	6	3462	3417	169	2974	2991	2974	79	5127
NASD	1314	7666	1	4	3528	3541	173	2931	2860	2931	67	4971
RTS	2121	20219	43	42	12377	12323	6804	23156	23145	23156	116	16470
S&P500	1359	8702	4	8	4109	4053	170	3516	3513	3516	68	5889
DAGS	1265	5070	61	74	4961	5010	188	2849	2870	2849	401	6631
IBEX	1275	5524	129	146	6026	6066	289	3927	3927	3927	367	6685
MI30	1416	9091	215	177	8579	8581	212	5470	5476	5470	414	9415
CA40	1321	7242	55	71	6823	6914	224	4171	4191	4171	395	8192
Average	463	1999	64	66	6233	6238	1029	6124	6122	6124	77	1775

Table 7.1: LPM's for the different hedging approaches when including both forwards and options in the hedge. For each approach both LPM_ϕ and LPM_ρ is presented The values of the LPM's are quoted as 10^{-5}. The sub index SB indicates the result of the portfolio when re-hedged according to section 4.4.

7.2 The effects on the tail

Table 7.2 shows that using $\beta^{*\phi}_{f,o}$ in the stress test scenario resulted in almost opposite results to those of the in-sample. The differences of CVaR between the unhedged and hedged portfolios were in general larger here than for the in-sample data. An interesting result was that currency hedging had such a large impact on the portfolios' CVaR's. For a SEK investor in the Dow Jones Index there was an improvement in CVaR for the portfolio by 34 percentage points during the stress test period for $\beta^{*\phi}_{f,o}$.

EURSEK			USDSEK			EURUSD		
Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)
OMRX Bond	-4	-3	OMRX Bond	-18	16	Dow Jones	-21	15
OMXS30	6	-13	OMXS30	4	-4	NASDAQ	-16	0
DAX	-11	25	Dow Jones	-34	18	RTS	-15	10
IBEX	-11	10	NASDAQ	-26	0	S&P500	-19	17
MI30	-12	45	RTS	-30	7	DAX	3	-4
CA40	-11	27	S&P500	-32	22	IBEX	11	-16
						MI30	11	-21
						CA40	6	-21

Table 7.2: The differences in $CVaR_{95\%}$ calculated on the stress test scenario between an unhedged portfolio and a portfolio using the hedge ratios optimised on the historical data for LPM_ϕ; $\beta^{*\phi}_{f,o}$. The corresponding correlations between the assets and currencies for the worst 5% results are also presented.

In Table 7.3 we clearly see how changing the correlation structure affected the portfolios' tails during the stress test period when using $\beta^{*\phi}_{f,o}$. It is most evident for a SEK investor in the assets denoted in EUR where we see an increase in CVaR by between 20 and 31 percentage points for the hedged portfolio compared to for the unhegded portfolio during the stress test period. Table 6.4 tells us why; the net hedge ratios (the sum $\beta^{*\rho}_{f}$ and $\beta^{*\rho}_{o}$) when LPM_ρ was minimised are for all of these assets -200%, which leads to a 200% larger currency exposure compared to for the unhedged portfolio. When we then reversed the correlations these exposures affected the negative tails of the portfolio results negatively. The same effect was observed for the RTS Index, both for SEK and EUR investors (for which we also had negative net hedge ratios). For the other currencies and assets (for which we still have positive net hedge ratios) the results between the in-sample and stress test scenario were much more stable. For a complete list of the CVaR's for the stress test scenario see appendix E.

EURSEK			USDSEK			EURUSD		
Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)	Asset	ΔCVaR (pp)	Tail Corr 5% (%)
OMRX Bond	-3	-3	OMRX Bond	-16	16	Dow Jones	-23	15
OMXS30	20	-13	OMXS30	-5	-4	NASDAQ	-18	0
DAX	30	25	Dow Jones	-36	18	RTS	8	10
IBEX	30	10	NASDAQ	-26	0	S&P500	-21	17
MI30	31	45	RTS	18	7	DAX	-3	-4
CA40	30	27	S&P500	-34	22	IBEX	1	-16
						MI30	2	21
						CA40	2	21

Table 7.3: The differences in $\text{CVaR}_{95\%}$ calculated on the stress test scenario between an unhedged portfolio and a portfolio using the hedge ratios, $\beta^{*\phi}_{f,o}$, optimised on the historical data for LPM_ρ. The correlations between the assets and currencies for the worst 5% results are also presented

7.3 The effect of including options

EURUSD								
	min LPM$_\phi$				min LPM$_\rho$			
	Forwards	Forwards & Options		Improvement	Forwards	Forwards & Options		Improvement
	$\beta^{*\phi}_{f,f}(\%)$	$\beta^{*\phi}_{f}(\%)$	$\beta^{*\phi}_{o}(\%)$	(%)	$\beta^{*\rho}_{f,f}(\%)$	$\beta^{*\rho}_{f}(\%)$	$\beta^{*\rho}_{o}(\%)$	(%)
OMRX Bond	109	107	2	17	52	31	25	0
OMXS30	99	99	1	1	200	200	0	0
DAX	62	61	5	29	-200	-163	-37	1
IBEX	61	59	12	43	-200	-163	-37	1
MI30	52	50	16	46	-200	-200	0	0
CA40	60	59	8	41	-200	-163	-37	1

USDSEK								
	min LPM$_\phi$				min LPM$_\rho$			
	Forwards	Forwards & Options		Improvement	Forwards	Forwards & Options		Improvement
	$\beta^{*\phi}_{f,f}(\%)$	$\beta^{*\phi}_{f}(\%)$	$\beta^{*\phi}_{o}(\%)$	(%)	$\beta^{*\rho}_{f,f}(\%)$	$\beta^{*\rho}_{f}(\%)$	$\beta^{*\rho}_{o}(\%)$	(%)
OMRX Bond	106	102	7	0	81	52	42	0
OMXS30	71	62	28	32	69	-11	189	57
Dow Jones	72	59	50	-997	19	-27	173	75
NASDAQ	66	48	52	-49	-3	-43	157	68
RTS	35	16	111	-110	-129	-137	63	23
S&P500	69	56	52	-850	13	-30	170	73

EURUSD								
	min LPM$_\phi$				min LPM$_\rho$			
	Forwards	Forwards & Options		Improvement	Forwards	Forwards & Options		Improvement
	$\beta^{*\phi}_{f,f}(\%)$	$\beta^{*\phi}_{f}(\%)$	$\beta^{*\phi}_{o}(\%)$	(%)	$\beta^{*\rho}_{f,f}(\%)$	$\beta^{*\rho}_{f}(\%)$	$\beta^{*\rho}_{o}(\%)$	(%)
Dow Jones	83	68	34	0	50	-28	172	37
NASDAQ	82	72	23	0	52	-25	175	38
RTS	46	22	87	-4304	-128	-133	67	17
S&P500	80	66	36	0	52	-27	173	35
DAX	65	53	37	50	0	-37	163	28
IBEX	84	77	18	22	69	0	200	32
MI30	81	74	25	20	37	-12	188	32
CA40	75	66	29	38	25	-23	177	31

Table 7.4: Hedge ratios when using only forwards as well as hedge ratios using both forwards and options and the improvement in LPM by introducing options, calculated on the stress test scenario with reversed correlations during the financial crisis. The values of the β's presented are the optimal once for the historical data.

In table 7.4 we see that it is in most cases worth including options in the hedge even when the hedge levels are stress tested, both if we want a low LPM$_\phi$ or a low LPM$_\rho$. There are clear improvements in LPM$_\rho$ when using $\beta^{*\rho}_{f,o}$ for both EURUSD and USDSEK, and the results are similar to those of the in-sample data. When using the $\beta^{*\phi}_{f,o}$ we see large improvements compared to the unhedged portfolio in LPM$_\phi$ in most cases for the stress test period. However there are increases in LPM$_\phi$ for five assets (SEK investor in Dow Jones, NASDAQ Composite Index, RTS and S&P500 and for a EUR Investor in RTS) when including options in the hedge. Studying the table a bit further we see that this is where we have the largest positions in options. In figure 7.1 histograms for ϕ for these five assets are presented, both when including options in the hedge and when not.

As can be seen in the histograms there are large spikes in ϕ for the lowest values when using both options and forwards. This means that we will have a fairly certain small loss (around 2-5%) when including options compared to a more spread out and less probable negative side when using only forwards. Even though we clearly see that we have fatter tails for two of the assets (SEK investor in NASDAQ and SEK investor in RTS) when only using forwards LPM_{ϕ} is lower than when including both options and forwards in the hedge.

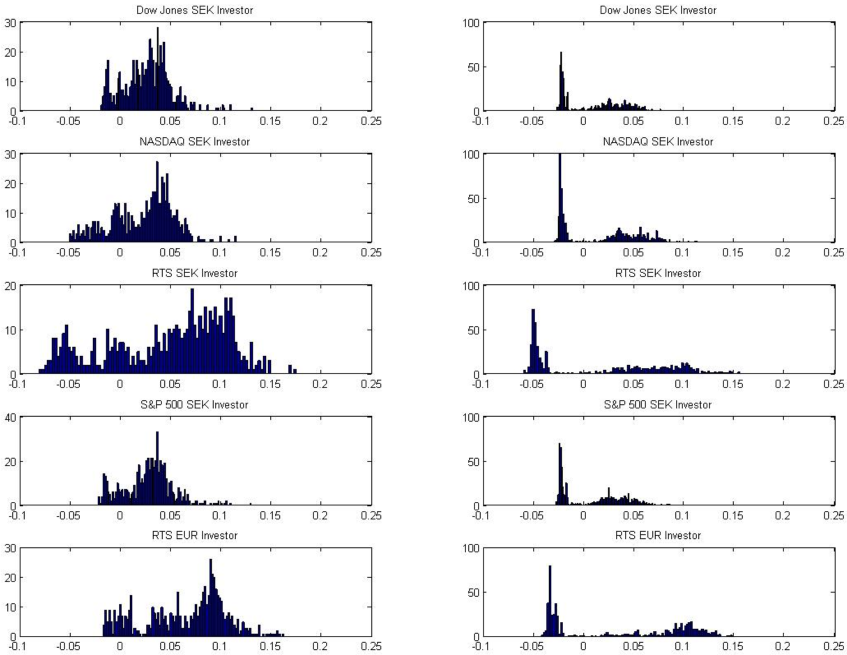

Figure 7.1: Histograms of ϕ for a SEK investor in Dow Jones, NASDAQ Composite Index, RTS and S&P 500 and for a EUR Investor in RTS, including only forwards in the hedge (left) and using both options and forwards (right).

Chapter 8

Conclusions

8.1 Discussion

This thesis investigates the effects of different hedging strategies using different measures for the success of the hedge, P and Φ (see Chapter 4 Models), depending on the preferences of the investor. Lower partial moments is used as a risk measure and data from the EUR, SEK and USD regions is used.

A main area of this thesis is the intentions of two different investors; one that believes in optimising a portfolio containing the underlying asset and hedging instruments, and one that wants to minimise any effects of changes in the future foreign exchange rate (avoid taking a speculative position in foreign currency). The first investor wants a small P and the second one wants a small Φ. The results of the optimisations and stress tests performed show that it might be more rewarding to focus on minimising Φ. One reason for this is that Φ can be affected to a much larger extent than P by choosing the hedge levels appropriately. Another reason is the sensitivity to changes in the correlations between the underlying asset and the currency. Minimising LPM_P means using the correlation structure between the asset and the currency to minimise the downside risk of the portfolio. To do this one should be certain that the optimal hedge levels obtained are stable to changes in the correlations. If they are not, a change in the correlation structure can have largely negative impacts on LPM_P. Also, the method involves taking a speculative position in a foreign currency. The hedge is hence based heavily on assumptions of the behaviour of macro variables. This is very different from the portfolio of the second investor for which we try to remove any dependency on the foreign currency (or on any variables other than the underlying asset). The results from the stress tests conducted also show that using the approach of minimising LPM_P can lead to very poor results when the properties of the market change. Of course, minimising LPM_Φ affects the tail of the entire portfolio. The results show both positive and negative effects depending on the correlations between the asset and the currency. We also note that the positions in forwards and options that optimise LPM_P are rather extreme and we believe that it is worth considering if using currency exposure is a reasonable way to hedge the worst outcomes of the underlying asset, or if maybe for example using out of the money options for the asset, is a better alternative.

Another result found is that it is (almost) always profitable to include options in the hedge, no matter what kind of investor you are. Another result yet again, is that the highest optimal level of options (when minimising LPM_Φ) is found for the most volatile asset and vice versa. Also, volatile currency pairs (USDSEK and EURUSD) have higher optimal hedge levels in options than the less volatile currency pair (EURSEK). Also, for EURSEK, the improvements are small when introducing options. These results are not surprising since a high hedge level in forwards can be costly if the underlying asset drops drastically in value. This is more likely to happen for volatile assets and the cost if it happens is more likely to be high for more volatile currencies.

We do not see any positive effects on neither P or Φ when testing the method of lowering the position in forwards when the implied volatilities for the currencies increase. Maybe it would be more interesting if we could instead reduce the hedge when correlations change, but values for the correlations are much more difficult both to calculate and to find on the market. The method of using fixed 80%-20% hedge levels shows results rather similar to the strategy of minimising LPM_Φ and could therefore be a good option for an investor striving to have a low Φ. Leaving the portfolio unhedged, on the other hand, will give very poor results for this investor.

Of course, the results discussed are based on rather few assets and currency pairs and should not be seen as absolute but rather as an indication of what the market looks like. On the other hand, the results fit rather well with what could be expected intuitively. The situation of an investor holding only one asset (or a combination of assets identical to those provided by the indices used) is not likely in real life but a simplification of a more complicated reality. We hope that this thesis will improve the readers understanding of what factors should be considered when deciding how to hedge a foreign investment.

8.2 Further Studies

In this thesis the focus has been an investor that invests in one foreign asset or index. A natural extension would be to increase the size of the investor's portfolio and allow it to consist of several foreign assets denoted in different currencies. For example one could use a Swedish investor that invests equal parts in assets denoted in EUR, USD and GBP respectively. This would be a more complicated situation since we would have more correlations to consider. This might severally affect the results when minimising the downside of the total portfolio. However the effects for an investor who wants to avoid currency exposure is much more unclear and hence the results would be of high interest. We also believe that the dynamics between the optimal hedge ratios and characteristics of the assets and currencies are interesting topics. In this study we have seen tendencies that more volatile assets and currencies should be hedged with larger levels of options, but we believe that deeper studies of this phenomenon could give more insight into the subject. One might also consider more advanced strategies for re-hedging; perhaps several different triggers as well as trends could be used to construct re-hedging schemes that could further increase the effectiveness of the currency hedges.

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Appendix A

Complete list of hedge levels

	min LPM_ϕ						min LPM_ρ					
	Forwards	Forwards SB	Forwards & Options		Forwards & Options SB		Forwards	Forwards SB	Forwards & Options		Forwards & Options SB	
	$\beta^{*\phi}_{f,f}(\%)$	$\beta^{*\phi}_{f,fSB}(\%)$	$\beta^{*\phi}_{f}(\%)$	$\beta^{*\phi}_{o}(\%)$	$\beta^{*\phi}_{fSB}(\%)$	$\beta^{*\phi}_{oSB}(\%)$	$\beta^{*\rho}_{f,f}(\%)$	$\beta^{*\rho}_{f,fSB}(\%)$	$\beta^{*\rho}_{f}(\%)$	$\beta^{*\rho}_{o}(\%)$	$\beta^{*\rho}_{fSB}(\%)$	$\beta^{*\rho}_{oSB}(\%)$
OMRX Bond	106	106	102	7	101	8	81	81	52	42	53	42
OMXS30	71	71	62	28	59	33	69	73	-11	189	-8	192
Dow Jones	72	69	59	50	55	53	19	17	-27	173	-28	172
NASDAQ	66	59	48	52	37	64	-3	-9	-43	157	-48	152
RTS	35	32	16	111	14	115	-129	-128	-137	63	-136	64
S&P500	69	66	56	52	50	58	13	9	-30	170	-31	169

	min LPM_ϕ						min LPM_ρ					
	Forwards	Forwards SB	Forwards & Options		Forwards & Options SB		Forwards	Forwards SB	Forwards & Options		Forwards & Options SB	
	$\beta^{*\phi}_{f,f}(\%)$	$\beta^{*\phi}_{f,fSB}(\%)$	$\beta^{*\phi}_{f}(\%)$	$\beta^{*\phi}_{o}(\%)$	$\beta^{*\phi}_{fSB}(\%)$	$\beta^{*\phi}_{oSB}(\%)$	$\beta^{*\rho}_{f,f}(\%)$	$\beta^{*\rho}_{f,fSB}(\%)$	$\beta^{*\rho}_{f}(\%)$	$\beta^{*\rho}_{o}(\%)$	$\beta^{*\rho}_{fSB}(\%)$	$\beta^{*\rho}_{oSB}(\%)$
OMRX Bond	109	103	107	2	95	11	52	52	31	25	31	25
OMSX30	99	75	99	1	75	0	200	200	200	0	104	96
DAGS	62	60	61	5	58	11	-200	-121	-163	-37	-78	-122
IBEX	61	57	59	12	56	11	-200	-136	-163	-37	-99	-101
MI30	52	51	50	16	49	20	-200	-190	-200	0	-158	-42
CA40	60	58	59	8	57	11	-200	-143	-163	-37	-98	-102

	min LPM_ϕ						min LPM_ρ					
	Forwards	Forwards SB	Forwards & Options		Forwards & Options SB		Forwards	Forwards SB	Forwards & Options		Forwards & Options SB	
	$\beta^{*\phi}_{f,f}(\%)$	$\beta^{*\phi}_{f,fSB}(\%)$	$\beta^{*\phi}_{f}(\%)$	$\beta^{*\phi}_{o}(\%)$	$\beta^{*\phi}_{fSB}(\%)$	$\beta^{*\phi}_{oSB}(\%)$	$\beta^{*\rho}_{f,f}(\%)$	$\beta^{*\rho}_{f,fSB}(\%)$	$\beta^{*\rho}_{f}(\%)$	$\beta^{*\rho}_{o}(\%)$	$\beta^{*\rho}_{fSB}(\%)$	$\beta^{*\rho}_{oSB}(\%)$
Dow Jones	83	80	68	34	64	38	50	48	-28	172	-27	173
NASDAQ	82	75	72	23	63	29	52	43	-25	175	-27	173
RTS	46	44	22	87	20	89	-128	-126	-133	67	-131	69
S&P500	80	77	66	36	61	41	52	48	-27	173	-27	173
DAGS	65	63	53	37	52	34	0	10	-37	163	-34	166
IBEX	84	79	77	18	74	13	69	69	0	200	0	200
MI30	81	78	74	25	72	18	37	35	-12	188	-12	188
CA40	75	72	66	29	65	23	25	26	-23	177	-22	178

Table A.1: All the different optimal hedgeratios for approach 1, 2, 3 and 4. The subindex *SB* refers to Sell Back strategi

Appendix B

LPM's when only using forwards

EURSEK												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	395	70	0	15	1127	1	0	72	0	0	84	3
OMXS30	511	5390	2	12	2523	1	68	72	2929	3426	85	4381
DAX	88	3249	5	17	3830	3652	207	184	2259	2670	77	4005
IBEX	77	2069	5	5	2594	2460	193	188	1095	1388	59	2765
MI30	85	3259	8	6	3842	3714	205	330	1716	2192	95	4120
CA40	84	3027	6	6	3628	3468	195	212	1861	2277	80	3843
AVERAGE	207	2844	4	10	2924	2216	145	176	1643	1992	80	3186

USDSEK												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	1353	653	2	3	9	8	59	54	1	1	244	40
OMXS30	1685	6297	59	63	4993	4857	60	64	4993	4856	312	5528
Dow Jones	1631	2210	113	135	2101	2142	599	634	1641	1654	404	2386
NASDAQ	1677	5182	173	236	5396	5422	972	1093	4502	4488	628	5917
RTS	1469	3920	602	634	5137	5096	3526	3494	1360	1257	954	5478
S&P500	1666	3021	125	158	2959	3036	695	757	2397	2412	440	3356
AVERAGE	1580	3547	179	205	3432	3427	985	1016	2482	2445	497	3784

USDEUR												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
Dow Jones	1268	2294	53	63	1700	1712	171	186	1637	1644	311	1983
NASDAQ	1340	5185	68	101	4313	4373	164	226	4257	4300	424	4936
RTS	1212	4151	378	395	4707	4700	2966	2957	2999	2975	697	4855
S&P500	1293	3061	59	72	2348	2371	154	180	2301	2315	339	2741
DAX	723	3883	59	51	3493	3470	362	281	3289	3283	278	4067
IBEX	743	3140	15	17	2307	2274	27	23	2292	2265	166	2814
MI30	730	3818	32	30	3363	3364	114	111	3233	3238	230	3965
CA40	730	3657	43	38	3214	3190	188	171	3044	3037	240	3799
AVERAGE	1005	3649	89	96	3181	3182	518	517	2882	2882	336	3645

Table B.1: Complete list of LPM's when using only forwards for hedging. For each approach we calculate both LPM_ϕ and LPM_ρ is presented. The values of the LPM's are quoted as 10^{-5}. The sub index *SB* indicates the result of the portfolio when re-hedged according to section 4.4.

Appendix C

Complete list of CVaR's

Asset	No hedge			min LPM$_\phi$			min LPM$_\rho$		
	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$ (%)	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$ (%)	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$
OMRX Bond	-9	-11	-13	-1	-2	-2	-1	-1	-1
OMSX30	-52	-56	-59	-43	-47	-51	-43	-48	-52
DAGS	-50	-56	-58	-48	-53	-56	-49	-55	-57
IBEX	-38	-43	-47	-42	-47	-51	-33	-37	-40
MI30	-44	-48	-50	-49	-54	-57	-36	-43	-46
CA40	-43	-47	-50	-43	-46	-49	-41	-46	-49

Asset	No hedge			min LPM$_\phi$			min LPM$_\rho$		
	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$ (%)	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$ (%)	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$
Dow Jones	-37	-43	-46	-40	-46	-52	-28	-33	-36
NASDAQ	-57	-64	-66	-52	-57	-62	-52	-57	-61
RTS	-70	-74	-75	-71	-75	-76	-51	-61	-66
S&P500	-41	-45	-48	-43	-49	-54	-32	-37	-40
DAGS	-51	-56	-58	-46	-55	-60	-37	-41	-43
IBEX	-52	-59	-61	-37	-41	-43	-33	-38	-42
MI30	-58	-65	-67	-42	-46	-48	-38	-44	-48
CA40	-53	-57	-59	-40	-44	-49	-35	-40	-43

Asset	No hedge			min LPM$_\phi$			min LPM$_\rho$		
	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$ (%)	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$ (%)	CVaR95% (%)	CVaR$_{99\%}$ (%)	CVaR$_{99.9\%}$
OMRX Bond	-24	-30	-34	-3	-4	-4	-1	-2	-3
OMXS30	-59	-62	-65	-49	-54	-56	-48	-55	-58
Dow Jones	-39	-41	-42	-40	-46	-51	-24	-28	-32
NASDAQ	-55	-58	-62	-54	-59	-61	-49	-55	-58
RTS	-68	-71	-73	-70	-73	-75	-34	-42	-47
S&P500	-44	-46	-47	-42	-48	-53	-31	-38	-42

Table C.1: CVaR for ρ at 95%, 99% and 99.9% for appraoch 1, approach 2 and un hedged

Appendix D

LPM's when only using forwards, stress test

EURSEK												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	26	3	0	0	0	0	1	1	0	0	17	0
OMSX30	15	4356	143	22	5702	5714	2679	2814	10736	10890	36	5868
DAGS	1024	8216	13	17	4910	5174	10286	5122	18078	13308	86	5712
IBEX	1026	8391	20	26	5201	5448	10399	6035	18116	14243	101	6018
MI30	821	10808	5	5	7494	7974	10586	9795	19618	19064	52	8486
CA40	1048	9964	11	14	6349	6626	11009	6807	20385	16676	81	7265
AVERAGE	238	1662	32	14	4943	5156	7493	5096	14489	12363	22	1328

USDSEK												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
OMRX Bond	1362	770	0	0	0	0	36	35	0	0	200	17
OMSX30	1238	5512	84	226	5353	6168	77	239	5743	6221	170	6253
Dow Jones	3525	10846	2	4	2491	3983	1063	1031	7425	7356	161	5507
NASD	3503	10668	15	48	2826	4481	2334	2654	9141	9550	187	5502
RTS	5067	22860	50	71	8188	16143	52225	56235	40761	41770	173	16711
S&P500	3590	11774	2	6	2978	4748	1350	1366	8726	8739	155	6352
AVERAGE	1290	2755	25	59	3639	5921	9514	10260	11966	12273	74	1784

EURUSD												
	No hedge		min LPM_ϕ				min LPM_ρ				Fixed	
	LPM_ϕ	LPM_ρ	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	$LPM_{\phi SB}$	LPM_ρ	$LPM_{\rho SB}$	LPM_ϕ	LPM_ρ
Dow Jones	1328	7819	0	0	3417	3857	36	30	4760	4697	79	5127
NASD	1314	7666	0	0	3541	3871	34	61	4611	4742	67	4971
RTS	2121	20219	1	2	12323	15304	16467	16711	27815	27843	116	16470
S&P500	1359	8702	0	0	4053	4595	21	21	5402	5359	68	5889
DAGS	1265	5070	120	140	5010	5367	821	821	3989	3989	401	6631
IBEX	1275	5524	164	182	6066	6258	105	131	5794	5857	367	6685
MI30	1416	9091	268	220	8581	8912	297	321	8046	8015	414	9415
CA40	1321	7242	89	108	6914	7240	434	449	6094	6110	395	8192
AVERAGE	463	1999	80	82	6238	6926	2277	2318	8314	8327	77	1775

Table D.1: LPMs for the different approaches when only using forwards for hedging (except for the fixed strategy that still has 20% options) for the stress test scenario. For each strategi both LPM_ϕ and LPM_ρ is presented. The values of the LPM's are quoted as 10^{-5}. The sub indices SB indicates the result of the portfolio when re-hedged according to section 4.4.

Appendix E

Complete list of CVaR's, stress test scenario

Asset	No hedge			min LPM_ϕ			min LPM_ρ		
	CVaR (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)
OMRX Bond	-2	-3	-4	2	1	1	0	0	-1
OMSX30	-42	-46	-48	-49	-53	-55	-62	-65	-67
DAGS	-53	-55	-56	-42	-44	-46	-83	-90	-91
IBEX	-55	-58	-59	-44	-47	-49	-85	-92	-94
MI30	-62	-65	-67	-50	-52	-53	-93	-100	-102
CA40	-55	-57	-58	-43	-45	-47	-85	-91	-93

Asset	No hedge			min LPM_ϕ			min LPM_ρ		
	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	CVaR99.9 (%)
Dow Jones	-55	-60	-61	-34	-37	-38	-33	-35	-37
NASDAQ	-55	-58	-62	-39	-42	-45	-37	-40	-41
RTS	-81	-83	-83	-66	-68	-70	-90	-92	-94
S&P500	-57	-62	-63	-38	-40	-43	-36	-39	-40
DAGS	-40	-42	-43	-43	-46	-49	-36	-40	-42
IBEX	-40	-43	-44	-51	-54	-55	-41	-44	-45
MI30	-48	-50	-51	-59	-63	-64	-47	-48	-49
CA40	-41	-44	-45	-47	-49	-53	-39	-42	-43

Asset	No hedge			min LPM_ϕ			min LPM_ρ		
	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)	$\text{CVaR}_{95\%}$ (%)	$\text{CVaR}_{99\%}$ (%)	$\text{CVaR}_{99.9\%}$ (%)
OMRX Bond	-15	-16	-16	4	4	3	1	1	1
OMXS30	-43	-45	-46	-47	-51	-54	-38	-40	-43
Dow Jones	-64	-68	-70	-30	-32	-35	-28	-31	-35
NASDAQ	-63	-66	-68	-37	-38	-40	-37	-38	-39
RTS	-85	-87	-87	-56	-57	-58	-103	-108	-109
S&P500	-65	-70	-72	-34	-36	-37	-31	-34	-37

Table E.1: CVaR for ρ at 95%, 99% and 99.9% for approach 1, approach 2 and un hedged for the stress test scenario